EVALUATION OF CRITERIA FOR OPTIMAL AQUIFER PARAMETERS

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To my parents

SARDAR BALWANT SINGH

&

Smt. PYAR KAUR

Submitted on 6/8/85

CERTIFICATE

Certified that the work presented in this thesis entitled, 'Evaluation of Criteria for Optimal Aquifer Parameters' by Sri Mahendra Singh has been carried out under my supervision and it has not been submitted elsewhere for a degree.

6/8/85

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Mahendra Singh

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LIST OF SYMBOLS

SYMBOL		DESCRIPTION
V	=	Superficial Velocity of water in aquifer
k	=	Permeability
i	=	Hydraulic gradient
T	=	Coefficient of transmissivity
S	=	Coefficient of storativity
σ	=	Specific conductivity of a medium
$h_{i,j}$	=	Head in cell (i,j)
E	=	Voltage
I_e	=	Current
A	=	Area of cross section through which water flows
B	=	Saturated thickness of the aquifer
Index k	=	Value of observation at time instant k
Index $k+1$	=	Value of observation at time instant $k+1$
Index $k+1/2$	=	Value of observation at mid point between k and $k+1$ time instant
$r_{i,j}^{k+1/2}$	=	Recharge rate per unit area in the cell(i,j) during period at between times k and $k+1$
$p_{i,j}$	=	Pumping rate per unit area in the cell(i,j) during period t between times k and $k+1$
$a_{i,j}^{k+1/2}, b_{i,j}^{k+1/2} \ldots f_{i,j}^{k+1/2}$	=	LHS coefficients of the balance equation

Note: Other symbols are explained, where-ever they appear.

ABSTRACT

Parameter identification is one of the most important problems in groundwater system analysis. For a local problem the aquifer parameter can be found by conducting pumping tests. But for a regional problem the pumping test method will be too expensive. A method using the multicell approach is studied to find the aquifer parameters for a region. The problem can be stated as an optimization problem, linear or non-linear. The ground water flow is represented in the form of equations, called as balance equations which are obtained from the law of continuity and Darcy law. The equations are converted into linear constraints to solve the optimization problem. Several criteria can be thought of in formulating the objective function. The basic principle to form the criteria is to minimize the discrepency occuring between two sides of the flow equation. Criteria used to formulate the objective function are linear criteria A,B,C,D. In criterion E the square of the discripencies is considered. Using these criteria an optimal set of values of aquifer parameters are found. A method to modify the objective function for criterion E is also explained under the name of 'Modified Optimization' to get improved results. The idea behind the method is to use the first estimates or approximate values of parameters so that the results are close to the field values.

The data used for different cells do not have equal reliability. Hence weights are assigned depending upon the number of observations in a cell. The method of applycing the weights has also been explained.

All criteria are applied to the aquifer of different shapes with different boundary conditions. The results obtained are discussed to evaluate the criteria.

CHAPTER - 1

INTRODUCTION

1.1 GENERAL:

In olden times ~~the~~ societies used perennial streams as source of water supply. With time the demand of water increased and they searched for other alternatives and learned the use of groundwater as source of water supply. New techniques were developed. These include forecasting, evaluation, assessment of groundwater etc. All these have great importance to society and have wide application in field. The groundwater table may be needed within certain limits for a number of purposes. Forecasting helps in design~~ing~~ and construction of water-resources systems providing measures for regulating the groundwater regime in irrigation areas. In agriculture ~~the~~ forecasting is very much important since the availability of water to the plants through their roots from groundwater depends upon depth of water table. Also if the water table is too high it may cause water logging and loss of crops. Effect of pumping, recharge or groundwater flow has to be known to forecast the groundwater table.

To keep the restriction on the groundwater table or piezometric surface for different purposes, several constraints are imposed on certain aspects like pumpage or recharge. The

management problem can not be solved untill we know the behaviour of the system.

1.2 AQUIFER AND ITS PROPERTIES:

The permeable geologic formations which can store and transmit water are termed as aquifers. The formation is considered permeable if the rock material contains cracks, fissures and interconnected pores through which the water can flow. From analytical point of view the properties of the aquifer can be divided into microproperties and macro-properties.

1.2.1 Microproperties:

These properties of the aquifer can be determined from a sample taken from it and testing it in laboratories. These are porosity, specific yield and permeability. Work on porosity has been done by Terzaghi [1948], Muskat [1946], and Luthin [1966].

Specific yield is a measure of water yielding capacity. Methods for determining specific yield were outlined by Meinzer [1959]. Though specific yield is a microproperty it is equal to the coefficient of storage of an unconfined quifer, which is a macroproperty.

Permeability of an aquifer is a measure of its ability to transmit liquid through its network of pores. According to

Darcy's law

$$V = ki \tag{1.1}$$

where, V = Superficial discharge velocity

k = pearmeability

i = hydraulic gradient.

Specific velocity $V_s = V/n$ where n is the porosity. The coefficient of permeability varies more than any other engineering property of the aquifer. It can be measured by constant head method[For relatively permeable material] or falling head test [for low permeability], Fishel [1942]. Values of k are different for different directions. Generally its value for horizontal direction is more than the value for vertical direction.

1.2.2 Macroproperties of Aquifers:

These are those properties which can be determined for the aquifer as a whole from field tests. Laboratory studies can yield information on porosity, specific yield, and coefficient of permeability of a sample. The permeability variation within the aquifer can be estimated by taking a number of samples taken at different points in the aquifer. However, quite often the average values obtained are not indicative of the true condition of the aquifer. To find the permeability of the aquifer field tests are conducted which are also known as performance tests. With the help of

the field tests the aquifer macroproperties: transmissivity or coefficient of transmissibility (T) and storativity or coefficient of storage (S) are measured. These coefficients are defined as follows:

Transmissivity is defined as rate of flow of water (m^3/day) through a vertical strip of aquifer of unit width and extending the full saturation height under unit hydraulic gradient [Theis,1935]. Value of T is equal to k multiplied by the thickness of the aquifer.

The storage coefficient S is defined as the volume of the water the quifer releases or losses from storage per unit surface area per unit decline of head. In unconfined aquifer the storage coefficient is nearly equal to the specific yield.

1.3 DETERMINATION OF AQUIFER PROPERTIES FROM FIELD TESTS:

The aquifer performance tests may be steady state or unsteady-state test. The groundwater flow is represented in the form of mathematical model with certain assumptions and simplifications applied to the real flow. This model consists of mathematical equations which on solving subject to initial and boundary conditions give the parameters of the model. If the model is very near to the real system the values obtained can be treated as real values. The field tests provide data on pumping rate and draw-down at one or more observation wells.

From these data the aquifer parameters T and S can be calculated using equilibrium/nonequilibrium equations.

Equilibrium equation [Thiem,1906) is based on assumptions.

(i) Aquifer is homogeneous, isotropic and infinite in aereal extent.

(ii) The discharge well penetrates fully.

(iii) T is constant with time and space.

(iv) Pumping is constant for long time and steady state is reached.

(v) Flow is laminar

Value of T is given by

$$T = Q \frac{\log(r_2/r_1)}{2\pi\,(s_2-s_1)} \qquad (1.2)$$

where, Q = discharge at pumping well

r_2, r_1 = radial distances of observation wells from the pumping well

s_2, s_1 = draw-down in observation wells after the steady state has reached.

The non-equilibrium equation was presented by Theis [1935] and modified by Jacob [1950]. The assumptions are:

(i) Well has infinitesimal diameter ;

(ii) water removed from storage is discharged instantaneously with decline in head ;

(iii) assumptions (i), (ii), (iii) and (v) as in equilibrium equation.

The non-equilibrium equation is a solution of second order differential equation subject to the boundary conditions specified above. The equation is written as:

$$s = \frac{Q}{4\pi T} \int_{r^2S/4Tt}^{\infty} \frac{e^{-u}}{u} du \qquad (1.3)$$

where, s = drawdown

t = time after pumping started

$u = \dfrac{r^2S}{4Tt}$

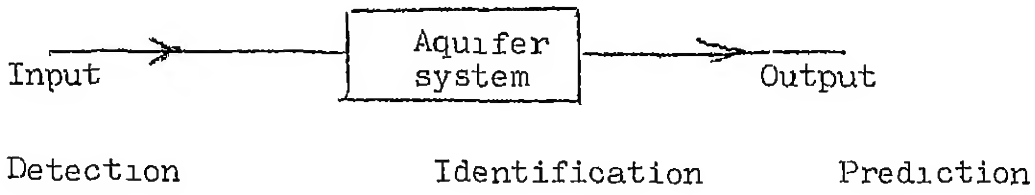
The data from the pumping tests are analyzed using the analytical model selected; usually presented graphically in the form of type curves. The detailed description of aquifer test equation was presented by Hantush [1964]. As stated already a model is an approximate representation of the real system. It can never represent the system exactly. Hence exact results for real situations can never be obtained. However a properly conducted pumping coupled with careful analysis can lead to good estimates of the parameters of the system.

1.4 SYSTEM APPROACH:

The pumping test methods are confined to a small area as they give local information only. They can not be applied

to the whole of the basin. It is too expensive to conduct the pumping tests all over the basin. Moreover the assumptions implicit in these tests may not be consistent over the extant of a basin. For a regional problem where the area is very large other approaches are used. A general review of this problem known as inverse problem or some times referred to as 'Calibration of the aquifer model' is given by Kisiel and Duckstein [1972].

Consider a system configuration which has input as known information during past, output as the information needed in future.



The black box is the system which transform input into output. There are in general three types of problems:

(i) Detection problem

(ii) Identification or inverse problem

(iii) Prediction problem.

Detection involves determining the inputs to the system, given both the response of the system and the system outputs.

Prediction problem involves determining the output of the system, given inputs and the system. The problem has unique solution. The identification involves determining the parameters which govern the response of the system. This is an extremely important problem and generally the solution obtained is not unique. We have to choose those results which on fitting in groundwater model give output consistent with the observed outputs.

1.4 OBJECTIVE OF THE STUDY:

The objective of the study is to find out the quifer parameters by solving the identification problem. The problem is solved using different criteria. The results obtained by these criteria are compared. The effect of different boundary conditions on the criterion selected is also considered in the study.

1.5 ORGANISATION OF THE STUDY:

The study made is reported in following chapters.

Chapter 2 deals with literature survey. This includes theory of groundwater modelling, detailed study of identification problem and work done in the past.

Chapter 3 discusses the detailed theory of the approach adopted to solve identification problem.

Chapter 4 deals with the application of the theory studied in Chapter 3 to the cases considered.

Chapter 5 deals with the results, discussion of results and conclusions.

CHAPTER - 2

LITERATURE REVIEW

2.1 GROUNDWATER MODELLING:

To study the behaviour of the groundwater flow in a basin it is not possible to conduct the cause effect tests on the prototype. For this modelling of groundwater flow is used. This is the process of simulating the groundwater flow for given situation. A model represents the characteristics of its prototype. With the help of modelling techniques the behaviour of the basin can be studied at very low cost. The basic principles and types of models have been discussed in the following sections.

2.2 BASIC PRINCIPLES OF GROUNDWATER MODELLING:

Groundwater movement is governed by some hydrological principles. The basic principles are the Darcy law and continuity equation. Hagen [1839] and Poiseuille [1846] showed that the velocity of the flow in a capillary tube is proportional to the hydraulic gradient. Darcy [1856] observed that the velocity of flow through porous media (laminar flow) is proportional to the first power of the hydraulic gradient. This law represents the macroscopic equivalent of the Navier Stokes equation for the viscous flow of groundwater. Viscous effects are accounted for completely by Darcy law. Mathematically

it can be represented as

$$V = k I \tag{2.1}$$

where V = velocity of flow in porous media

k = coefficient of permeability

I = hydraulic gradient.

The continuity equation is represented as

$$\frac{\partial}{\partial x}(\rho V_x) + \frac{\partial}{\partial y}(\rho V_y) + \frac{\partial}{\partial z}(\rho V_z) + \frac{\rho q}{b} = S_s \frac{\partial \rho}{\partial t} \tag{2.2}$$

where V_x, V_y, V_z = velocity of flow in x,y,z directions respectively

ρ = mass density of fluid

b = saturated thickness of aquifer

q = $q(x,y,z,t)$, amount of water added or extracted/area of the system per unit time

S_s = coefficient of specific storage.

From the above two equations the differential equation governing the groundwater flow for two dimensional flow in quifer having uniform thickness can be obtained [Jacob,1950, Hantush,,1964, De Wiest ,1965] as

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = \frac{S}{T} \frac{\partial h}{\partial t} \tag{2.3}$$

where S = storage coefficient

= $S_s.b$

T = transmissivity

h = head.

The groundwater flow can thus be represented in the form of a differential equation and solution of these equations will explain the behaviour of the groundwater flow. The above equation can be used for solving the different types of the problems [Jacob,1950 ; Jacob and Lohman,1952 ; Hantush, 1964].

2.3 DIFFERENT TYPES OF GROUNDWATER MODELLING TECHNIQUES:

The types of models can be catagorized as porous media models; miscellaneous analog models; electrical analog models based on similarity between Ohm's law and Darcy's law, and digital computer models for numerical solution of aquifer flow equations.

A brief discussion on these models has been given in the following sections.

2.3.1 Porous Media Models:

(1) Sand Tank Models: The first groundwater model was used by P. Forchheimer. He constructued a sand tank model for study of well flow in Graz, Austria in 1898. This model is a scale model of the aquifer with scaled down boundaries and modified permeability. These are constructed in water tight

boxes of various shapes, like rectangular forms, columns, etc. Sand tank models have been widely employed for investigating a variety of groundwater flow problems, such as well flow, seepage, artificial recharge, dispersion, and sea water intrusion.

(2) Transparent Models: A transparent porous media model can be constructed by matching the index of refraction of the medium with that of fluid. The moment of the fluid can now be traced by means of dye streams.

2.3.2 Analog Models:

Flow through porous media obeys laws that governs other physical systems including laminar flow of fluids; heat, and electricity. Techniques were developed depending upon these similarities. Non-electrical analog models are described in this section.

(1) Viscous Fluid Model or Hele Shaw Model or Parallel Plate Models: The first viscous fluid model was developed by Hele Shaw [1897] to demonstrate flow patterns around variously shaped boundaries. This model uses the analogy between the groundwater flow and the movement of a viscous liquid flowing between two closely spaced parallel plates.

(2) Membrane Models: In this model the surface of groundwater is represented by stretched thin rubber membrane. A central probe, representing a pumping well, deflects the membrane. Deflections are analogous to drawdown and can be

measured by micrometer or by optical techniques.

(3) Thermal Models: A thermal analogy can be developed for studies of flow in homogeneous and isotropic confined aquifers. The flow of heat in a uniform body of materials satisfies the Laplace equation and hence moves on a potential flow system in the same manner as groundwater [Hansen,1952].

2.3.3 Electric Analog Model:

The flow of an electric current is expressed by Ohm's law as,

$$I_c = -\sigma \frac{dE}{dx}. \qquad (2.4)$$

where I_c is the current per unit area through a material of specific conductivity σ and dE/dx is the voltage gradient. Equation (2.4) satisfies the Laplace equation and when compared with Darcy's equation

$$V = -k \frac{dh}{dx} \qquad (2.5)$$

similarity between the two equations is evident. It can be seen that velocity V is analogous to electric current I_c, hydraulic conductivity k to specific conductivity σ, and head h to voltage E. Depending upon the similarity explained above the following types of the models can be created.

Continuous systems, for which aquifer properties are modeled by an electric conductive medium that is continuous

in space and discrete systems in which the aquifer properties are modeled by an assemblage of discrete electric elements forming a network. Following are examples of electric analog model.

(1) Conductive Liquid Models: Limited to two dimensional steady-state situations, the model is formed by an insulated tank filled with an electrolyte such as a dilute solution of copper sulphate. Boundaries of the tank are scaled to represent the aquifer boundaries. To create equipotential surfaces the copper electrodes are immersed in the tank. The models have been applied to investigate a variety of seepage conditions, drawdown near a well field, and regional groundwater flows [Debrine, 1970; Todd and Bear, 1975].

(2) Conductive Solid Models: These models are best adapted for studies of steady flow in confined aquifers. The principle is same as that of liquid conductive models. The only difference is that instead of the liquid medium, here the solid medium is used. The conducting paper cut to the slope of boundary or conducting paint applied to non-conducting solids are used for this model [Sherwood, 1963 ; Shestakov, 1968].

(3) Resistance-Capacitance Networks: These networks are typically employed to evaluate confined aquifers under non-steady, two dimensional flow conditions. Resistance-capacitance networks are versatile in that they can readily study a variety

of aquifer conditions as well as extensive aquifers requiring a large number of nodes. The technique can be extended to three dimensional problem. The limitation on application of the analogy involves non-linear conditions of varying transmissivity in unconfined aquifers and two fluid flow problems [Walton,1963 ; Skibitzke, 1963].

4. Resistance Networks: It has the same principle as that of resistance capacitance networks. It is composed of an electronic analyzer coupled to an analog model and the analog consists of array of resistors only. Capacitors are eliminated so no storage elements are included. These can be applied to steady state cases only [Bauwer, 1962; Hubert, 1968 and Baturic-Rubcic 1969].

2.3.4 Digital Computer Models:

With the developments of digital computers it has become possible to study any type of aquifer problem by means of digital computer model. Kleinecke [1971], Prickett [1973]. The computers are used to solve the mathematical models of the aquifers. The aquifer behaviour is represented in the form of differential equations. Solution to these equation gives the requisite information.

Two types of the model used on digital computer are

(i) Finite difference models

(ii) Finite element models.

(i) Finite Difference Methods: In this method the aquifer is divided into a grid and the flow associated with single zone of aquifer is analyzed. The flow equation is based on equation of continuity.

Inflow - outflow = change of storage

This relation along with Darcy law for the equation of motion yields

$$\frac{\partial}{\partial x}\left(T\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(T\frac{\partial h}{\partial y}\right) - Q = S\frac{\partial h}{\partial t} \qquad (2.6)$$

where T = transmissivity

S = storage coefficients

h = head

Q = net flow removed from aquifer.

The above equation is written in finite difference form and solutions on digital computer are obtained [Weber,1979; Pinder et al., 1970 ; Freeze , 1969 and Prickett, 1971].

(ii) Finite Element Methods: The finite element techniques involves solving a differential equation for groundwater flow by means of variational calculus. Guymon [1970], Neuman,S.P. [1971], Pinder [1977] , Pricket [1975]. Aquifer is subdivided into 'Finite element' of arbitrary shape and sizes. Parameters are kept constant for a given element. But they may vary from element to element. Solution of the equation is found by minimizing a variational function.

2.3.5 Hybrid Computer Models:

Some times the finite difference solution takes very long time to complete the iterations. To overcome this difficulty a combination of a digital model and a resistance network analog is used, and the model is called as hybrid computer model. [Morris, 1972 ; Vemuri, 1969]. This approach is advantageous for solving iteration-intensive problems such as non-steady flow in unconfined aquifers.

2.4 IDENTIFICATION PROBLEM:

Identification is one of the three basic problems of system analysis (i.e. Detection, Prediction, Identification). This is the problem in which the parameters of the system are predicted with the help of the given inputs and outputs of the system. The behaviour of the system is represented in the form of partial differential equations. The general review of this problem, known as inverse problem or some time referred to as 'Calibration' of the model is given by Kiesel and Duckstein [1972].

The various types of inverse problems are aimed at the determination, respectively of the following elements (a) parameters i.e. I.P. type 1; (b) Initial conditions, I.P. type 2 ; (c) Boundary conditions, I.P. type 3; (d) Input, type 4; (e) More than one of the above quantities, type 5.

Most of the reported work in the area of hydrology concerns I.P. type 1. The work on other types have been done

among others by : Lattes and Lions [1969] on type 2, Phillipson [1971] on type 3; Moench and Kisiel [1970] on problem close to I.P. type 4. Some of the central approaches to the I.P. type 5 in the aquifer equation context include those of Cole [1973] ,Emsellem and de Marsily [1971], Frind and Pinter [1973], Haimes et al. [1968], Nelson [1968], Neuman [1973] , Vemuri and Karplus [1969], Scarascia and Ponzini [1972) and Stallman [1956].

The solution of inverse problem is a powerful method for parameter identification. Solution of this problem tries to find out those estimates of the parameters which on putting in model reproduce results, closest to the observed aquifer response to the real problem. Inverse problem is generally used for identifying parameters only, but it can be used to get other information also, like boundary conditions, initial conditions etc. The method to solve the inverse problem can be divided into two groups i.e. indirect and direct [Neuman, 1973]. The former consists of a repetitive solution of the direct problem with successive parameter estimates to obtain the closest reproduction of the historic water table records. The modification in the aquifer parameter values at the end of each iteration can be based upon mathematically rigorous procedure or subjective reasoning. In the direct method of solving inverse problem, the parameters are treated as dependent variables and are solved for directly.

Cooley and Sinclair [1976] classified the parameter calculation methods in three catagories.

(1) Method that involve direct substitution of measured (observed) hydraulic head data into the equation assumed to approximately goverm groundwater flow followed by calculation of parameters, either directly or through minimization of some function of equation of residuals.

(2) The trial and error fitting of measured heads to heads calculated with assumed flow equations.

(3) Optimization methods that minimize the value of a selected objective function composed of some measure of the difference between observed and computed heads.

The first method falls under the catagory of direct method of solving inverse problem whereas the second and third methods fall under the category of indirect method of solving inverse problem.

2.5 SOLUTION TO THE INVERSE PROBLEM:

The methods and application of this methods is discussed here in brief.

2.5.1 Methods of Solving:

As stated above the inverse problem can be solved by indirect or direct method. The indirect method of solving inverse problem is trial and error method and consists of

(1) Assuming certain value for the unknown parameters.

(2) Solving the direct problem with this assumed value i.e. outputs are obtained.

(3) Comparing the results obtained in step 2 with the actual observation , and if two do not correspond within a certain limit then.

(4) With the help of a suitable (adjustable algorithm) changing the values of parameters in step 1.

(5) Repeating steps 2 to 4 until satisfactory similarity (using a criterion function) between the observed and the computed value is obtained.

In direct method the parameters are considered as unknowns, no initial estimation are made. The problem is solved with the help of optimization techniques (linear programming or nonlinear programming).

2.5.2 Criterion Functions:

Different criteria can be used for optimality If b is true parameter vector and β is the estimated one, then the criterion may be ,

(i) The minimization of some function of (β-b). Since b is unknown sufficient a priori knowledge is required for good results.

(ii) Minimization of some function or functional of $e = u - \bar{u}$ i.e. the difference between the measured out put (including noise) and the model out put. This error can be used because e can be made measurable, $u = u(b)$, $\bar{u} = \bar{u}(\beta)$.

(iii) The minimization of some functional containing the measurable process output and the estimates of the state vector and the parameter vector. Second criterion is widely used using least square method. The problem reduces to the variational problem of finding β so what the functional $J(\beta)$ is extremized

$$J(\beta) = u(b) - \bar{u}(b).$$

2.5.3 Work on Identification Problem:

Kleinecke [1971] uses linear programming to obtain optimum values of the aquifer parameters. He imposes a rectangular grid on the aquifer and discretizes it. The aquifer parameters are represented at nodes, surrounded by rectangular area. The criterion function preferred by Kleincke is to minimize Z where

$$Z = \sum_{i} \operatorname{Max} \left| x_i^k \right| \qquad (2.7)$$

where x_i = error at node i for time k.

Haimes et al. [1968] formulate the problem of estimating aquifer parameters with a non-linear criterion function and solve it with multilevel optimization. They consider a rather specialized case of a cluster of production wells in an infinite aquifer. The aquifer is divided into N wedge shaped homogeneous regions each enclosing a single well. A well is supposed to draw water only from the area

enclosed in the wedge, and thus the lines delineating the regions act as impervious boundaries for the well. If

Total number of wells = N

Then the values of parameters have to be found such that Z is minimized.

$$Z = \sum_{i=1}^{N} \sum_{k=1}^{M} (\bar{h}_{i,k} - h_{i,k})^2 \qquad (2.8)$$

where M = number of pressure observation

$\bar{h}_{i,k}$ = calculated pressure head at well i at time k

$h_{i,k}$ = observed pressure head at well i, at time k.

$\bar{h}_{i,k}$ is obtained in close form by the method of images.

Vemuri Karplus [1969] look upon the problem of parameter estimation of an unconfined aquifer as a control problem in distributed parameter systems and use a hybrid computer to solve it.

If

$$\frac{\partial}{\partial x}\left[T(x,y)\frac{\partial h}{\partial x}\right] + \frac{\partial}{\partial y}\left[T(x,y)\frac{\partial h}{\partial y}\right] + Q = S(x,y)\frac{\partial h}{\partial t} \qquad (2.9)$$

then the problem is to compute in a region R, the values of T and S and boundary of R such that the functional J is minimized.

$$J = \int_{t_i}^{t_{i+1}} \int_R [\bar{h}(x,y,t) - h(x,y,t)]^2 \, dR dt \qquad (2.10)$$

Method of subjective calibration was used by Gates [1974]. The method is effectively a trial and error procedure. A suitable numerical scheme is set up to solve the equation

$$\nabla^2 h + Q = \frac{S}{T} \quad \frac{\partial h}{\partial t} \tag{2.11}$$

The unknown parameters and the recharge and discharge are now assigned values subjectively. The equation is solved to obtain $\bar{h}$ at grid points. This calculated $\bar{h}$ is compared with the observed h at the same point. Any of the unknown parameters are changed subjectively (without any automated algorithm) to obtain a better fit between $\bar{h}$ and h at as many as grid points as possible. This is the simplest method but require considerable knowledge of aquifer to start with.

A definite improvement over the subjective calibration described above was made by Lovell et al . [1972]. A considerable amount of subjective information about the aquifer is still required in this method, but this is now quantified and used in a logical way.

Emsellem and de Marsily [1971] developed a method which they called 'An Automatic Solution for the Inverse Problem', to determine T for steady state flow in heterogeneous isotropic aquifer. T is determined as an average value over an area ; the area defining the scale T is determined in the process. They start out by assuming the entire area as homogeneous and find a value T such that the norms of the error (or the selected criterion function) in computed flow is minimum. Subsequently the area is subdivided into smaller portions and with the same criterion function, the best T value is calculated for each subarea. Process is terminated when

no further improvement in the criterion function takes place.

Nelson [1968] developed an energy dissipation method to evaluate permeability for a heterogeneous, isotropic medium. The drawback of method is assumption of zero storage coefficient. Chen and Seinfield [1972] discuss the estimation of spatially varying parameters in partial differential equations. Cannon [1964] solves a different kind of parameter identification problem. He considers heat flow in a homogeneous isotropic rod of unknown diffusivity. Jones [1963] also gives a method to solve for diffusivity when it is a function of time. The method depends upon a closed form solution of the flow equation and can not be applied to a regional scale. Yeh and Tause [1971] apply the quasi-linearization method as developed by Bellman and Kalaba [1965] to a rather idealized and local groundwater pumping problem. This method may be an alternative to the graphical type-curve method but is not applicable to the regional flow problem.

A direct method for the identification of parameter of non-homogeneous aquifer was developed by Sagar et al. [1975]. The method does not require iterative solution of the aquifer equation. The shape of the surface representing observed depended variables (i.e. head) is approximated from measured samples by means of various interpolation algorithms. Once the various derivatives of the dependent variable are approximate

The inverse problems reduces locally to the algebraic equations of small dimensions. Aquifer conditions of heterogeneity and anisotropy are amenable to this method.

A non-linear optimization method for aquifer parameters estimation was developed by Deepak Kashyap and Satish Chandra [1981]. This is a numerical scheme developed to identify storage coefficient transmissibility, recharge coefficient and orientation of principal permeability directions. The scheme is based upon the constrained minimization of sum of the squares of the residuces in the Boussinnesq equation.

Comparison model method to identify the aquifer transmissivities in non-homogeneous, Anisotropic aquifer in steady state flow was developed by Giansilvio Ponzini and Alfredo Lozej [1982]. The method find out the inter block transmissivities referred to the sides of the network block. The method is based on the comparison between the real gradients and the ones generated by a 'comparison model' whose initial transmissivity value is arbitrarily chosen and constant throughout the surveyed area. The minimum head anomaly criterion and the bottleneck criterion enable one to select a value or a set of values of initial transmissivity. An algorithm was developed to reach the final solution.

2.6 MULTICELL APPROACH:

In the multicell approach, the area is divided into cells. The properties of the cell are represented at the

centre of the cell (node). The balance equation is written for each cell for each time period. The solution of these equations will give the value of aquifer parameters. Optimization techniques (linear programming or quadratic programming) are used to reach the set of optimal values. The different criterion to reach the optimal value of the parameters are given below.

(i) Minimize the maximum absolute deviation at the nodes.

(ii) Minimize the sum of the maximum absolute deviations for all time periods.

(iii) Minimize the sum of the maximum absolute deviations for all cells.

(iv) Minimize the sum of the absolute deviations for all time periods for all cells.

(v) Minimize the sum of the square of the deviations for all cells for all time periods.

2.7 MODIFIED OPTIMIZATION METHOD:

Emsellem and de Marsily [1969] in their 'automatic solution to inverse problem' found that sometimes the results produced were unsatisfactory or physically meaningless. They thought this was due to source/sink $(p_{i,j}-r_{i,j})$ function.

However that is not the only reason why unrealistic results are obtained Frind and Pinder [1973] have elaborated more on intrinsical and mathematical features in the solution of inverse problem under steady state and found that the system

is highly sensitive to the hydraulic heads. In fact hydraulic heads intervene as gradients (difference of heads divided by distance) and small variation in head can produce large variation in gradient. This is specially true for the cases where the piezometric surface is relatively flat. So there are chances of getting eratic results due to ;

(i) Finite difference discretization.

(ii) Functional representation of hydraulic head.

(iii) Interpolation in space and time of field hydraulic head data in order to assign heads to nodes.

Kleinecke [1971] used linear programming to solve inverse problem. He used the criteria of

$$\text{Min} \quad Z \;=\; \sum_i \sum_j \sum_k \left| X_{i,j}^{k+\frac{1}{2}} \right| \tag{2.12}$$

Kisiel and Duckstein [1972] commenting on Kleinecke's approach stated that the main draw back of the method is that many of the T_{ij} and S_{ij} do not appear in the optimal solution, which means that these parameters will have to be zero for Z to be a minimum. There is no satisfactory explaination for this [add Kisiel and Duckestein], although Kleincke speculates that with a data base spread over a longer period of time this may not happen. Whatever the length data,however there is nothing in the linear programming algorithm that would ensure the presence of all parameters in the optimal solution. An improvement to Kleineckis approach was done by changing the

objective function as

$$\text{Min} \quad Z = \sum_{i} \sum_{j} \sum_{k} \left(X_{i,j}^{k+\frac{1}{2}} \right)^2 \qquad (2.13)$$

Although some improvement was there but still unrealistic results were present in the solution. Small changes in heads produced big changes in values of T and S whereas the system was comparatively insensitive to the source sink values.

Agustin Navarro [1982] suggested not to modify head values. But modification in objective function was made. And system was the same as used gy Kleincecke. The methedology selected was based on restricting the resulting values of T and S, so that they were not very different from the actual field measurements or office estimation of T and S. If the T^e_{ij} and S^e_{ij} were the first choice as field parameters, the modified function to minimize, F_{mod} was taken as

$$F_{mod} = \sum_{ijk}\sum\sum \left[X_{i,j}^{k+\frac{1}{2}}\right]^2 + d\left[\sum_{ij}\sum \left(1 - \frac{T_{ij}}{T^e_{i,j}}\right)^2 + \sum_{ij}\sum \left(1 - \frac{S_{ij}}{S^e_{i,j}}\right)^2\right] \qquad (2.14)$$

where d is an arbitrarily chosen parameter at d = 0, F_{mod} is same as Z. In this case the resulting $T_{i,j}$ and $S_{i,j}$ will produce a set of residuals that are minimum but $T_{i,j}$ and $S_{i,j}$ could not be similar to T^e_{ij} and $S^e_{i,j}$. With d different from zero, residuals $X_{i,j}^{k+1/2}$ will be higher than it is with d =0 and highest value of $X_{i,j}$ will happen when d is relatively big,

and $T_{i,j}$ and $S_{i,j}$ are very similar to $T^e_{i,j}$ and $S^e_{i,j}$.

Acceptability Criterion:

Between the bounds $T_{i,j}$ and $S_{i,j}$ similar to $T^e_{i,j}$ and $S^e_{i,j}$ from one side ($d \to \infty$) and $X^{k+1/2}_{i,j}$ minimum from the other ($d = 0$) there is a range where the practical solution have to be found. Acceptable values of $T_{i,j}$ and $S_{i,j}$ are those for which the residuals are minimum. To do this values of d is increased from 0,1,2..... And the set of values are checked. The values which are in practical range for minimum d may be selected as true values.

Further Improvements:

The objective can further be improved by assigning weights to terms in objective function depending upon the area of corresponding cell or numbers of observations wells etc. The weights which depend upon the reliability of the data in the specific cell have been explained in the next chapter.

CHAPTER - 3

METHODOLOGY

This chapter deals with the detailed theorey of the approach adopted in present work to solve the identification problem. The theory of cell approach, with extension to multicell model has been discussed in detail. The procedure for applying weights to terms in objective junction has also been explained.

3.1 SINGLE CELL MODEL:

The entire area of the basin can be represented as a single cell. It is a very approximate model and suitable for those conditions only where the change in the characteristics of the basin with respect to space are negligible.

Consider the basin given in the Fig. 3.1a and Fig. 3.1b, with impermeable boundaries. The river has a discharge Q. The entire area is considered as one cell with discharge Q and precipitation, artifical recharge etc. The following equation can be written for the above case.

$$t\,[A\,(N + R - P) - Q] = AS\,[\bar{h}_{t+\Delta t} - \bar{h}_t] \qquad (3.1)$$

The equation is called water balance equation.

where N = Natural precipitation (volume/unit area/unit time)

R = artificial recharge (Volume/unit area/unit time)

P = pumping (volume/unit area/unit time)

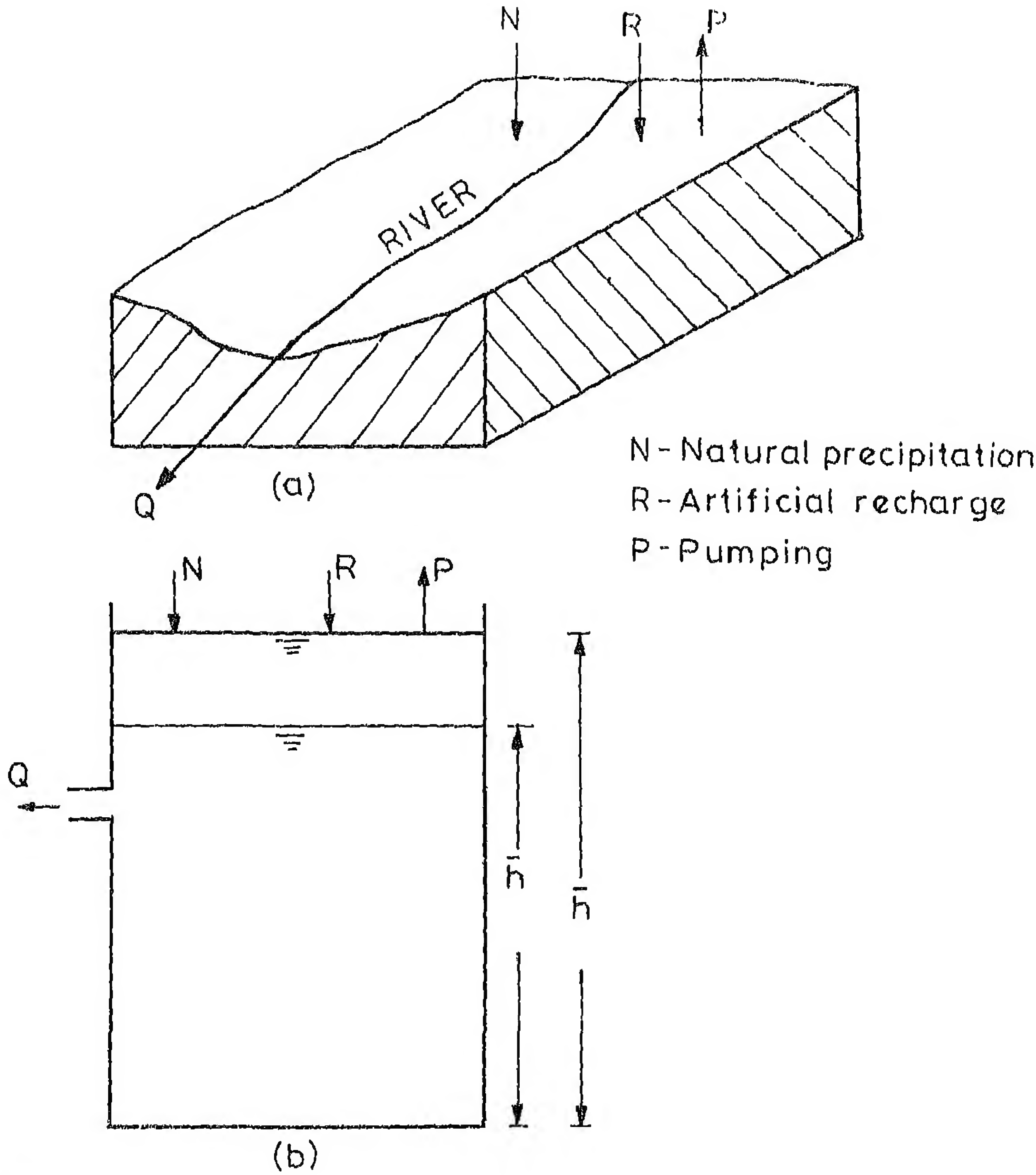


Fig.3·1 Single cell model of the aquifer

Q = outflow (volume/time)

$\bar{h}_t, \bar{h}_{t+\Delta t}$ = average water table elevation at time t and t + t respectively.

If the boundaries are impermeable, the flow that takes place through aquifer can be accounted for, by Darcy law. The area is divided into segments, as shown in Fig.3.2.

The volume of water flowing between two segments is given by Darcy law

$$Q = k.I.A$$
$$= k.I.\ w\ .\ b$$

where A = area of cross section through which the flow takes place

w = width of segment, perpendicular to the direction of flow

B = saturated thickness of the aquifer

Put in $T = k.b$

$$Q = T.I.w$$

Total inflow for the segments

$$= \sum_{i=1}^{N} T\ I_i\ w_i \qquad (3.2)$$

where N = number of segments.

Balance equation for a single cell with previous boundary

$$t[\ A\ (N + R-P) - Q + \sum_{i=1}^{N} T\ I_i\ w_i\ ]$$
$$= A\ S\ [\ \bar{h}_{t+\Delta t} - \bar{h}_t\ ] \qquad (3.3)$$

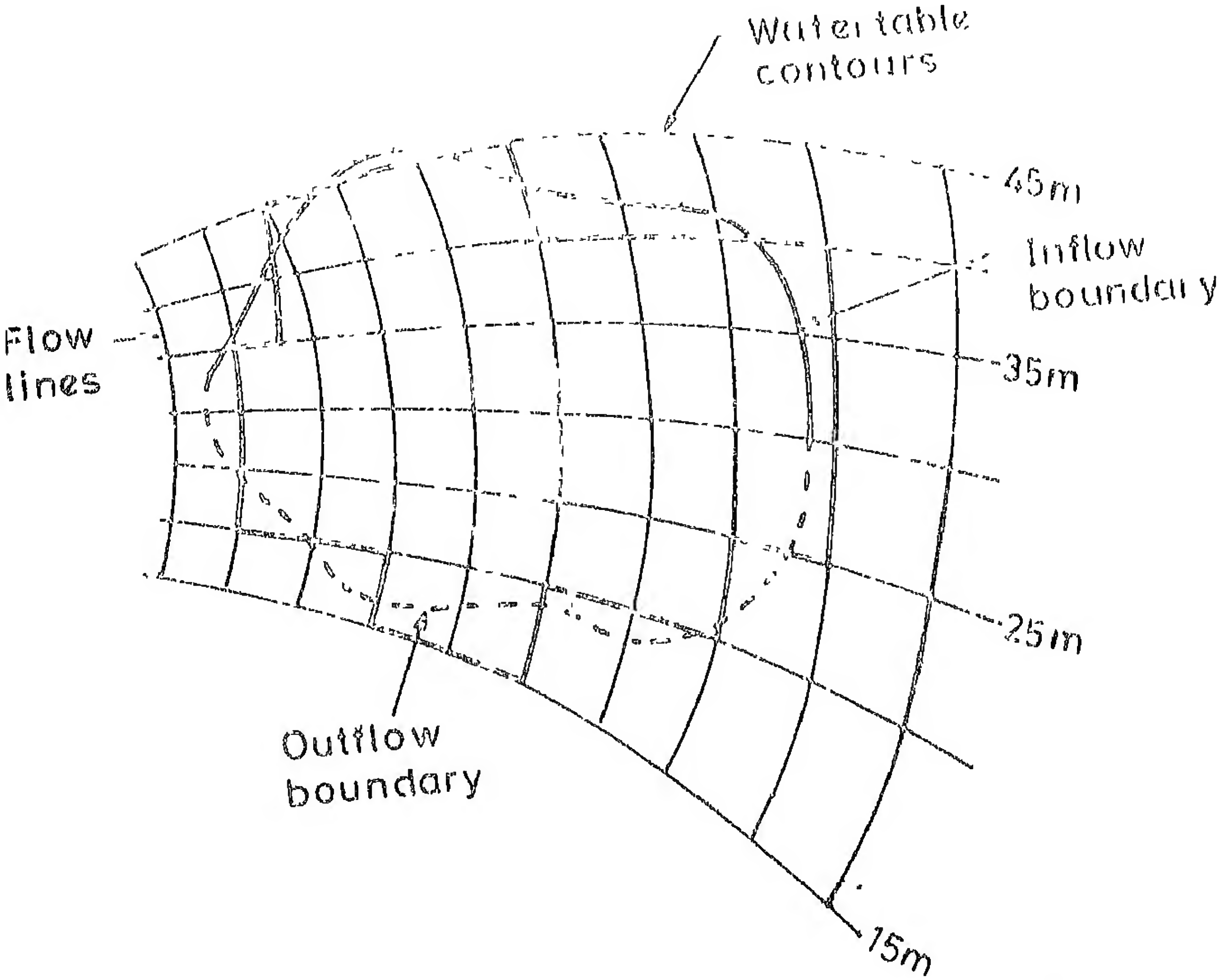


Fig.3·2 Flow through single cell in an aquifer

However this model is very approximate since there are spatial variations in aquifer characteristics.

3.2 MULTICELL APPROACH:

This approach is adopted if the area under consideration is very large, and there are appreciable variation in aquifer characteristics with respect to space.

Assumptions:

(i) The flow is in a confined aquifer or in a phreatic aquifer where spatial variations of water table are small with respect to the thickness of aquifer.

(ii) Flow is two dimensional.

(iii) Aquifer is isotropic but not homogeneous.

(iv) No flow takes place between two points within the same cell.

(v) The average properties of the cell are same within the cell.

The continuity equation is written as

$$\frac{\partial}{\partial x}\left[T\frac{\partial h}{\partial x}\right] + \frac{\partial}{\partial y}\left[T\frac{\partial h}{\partial y}\right] + r - p = S\frac{\partial h}{\partial t} \qquad (3.4)$$

where $T = T(x,y)$

$S = S(x,y)$

$r = r(x,y,t)$

$p = p(x,y,t)$.

r and p are replenishment and withdrawl from the aquifer/ unit horizontal area.

In multicell approach the entire area is discretized into a number of cells having regular areas (rectangle, square etc.). Consider an area shown in Fig. 3.3. The river has barrier boundaries on three sides and constant head boundary (river) on one side. The area has been divided into four cells. The average heads are shown in figure for different cells at a particular time.

In elevation the cross section of the cells is shown. Consider the small gates at the bottom (shown as opening). Gates are shown to indicate transmissibility.

If the gates are opened suddenly the flow will take place in the directions shown by arrows depending upon the head difference between two adjacent cells. The different characteristics of the cells such as T and S are defined at their center (nodes) Fig. 3.4 . Therefore the number of parameters to be found are twice the number of cells.

3.3 BALANCE EQUATIONS:

The finite difference form of the balance equation (3.4) is obtained in the following manner.

Discretize the whole area into rectangular cells (Fig.3.4). Let

$T_{i,j}$ = Transmissivity of cell (i,j)

$T_{i-1/2,j}$ = Transmissivity at the side dividing cells at (i,j) and (i-1,j)

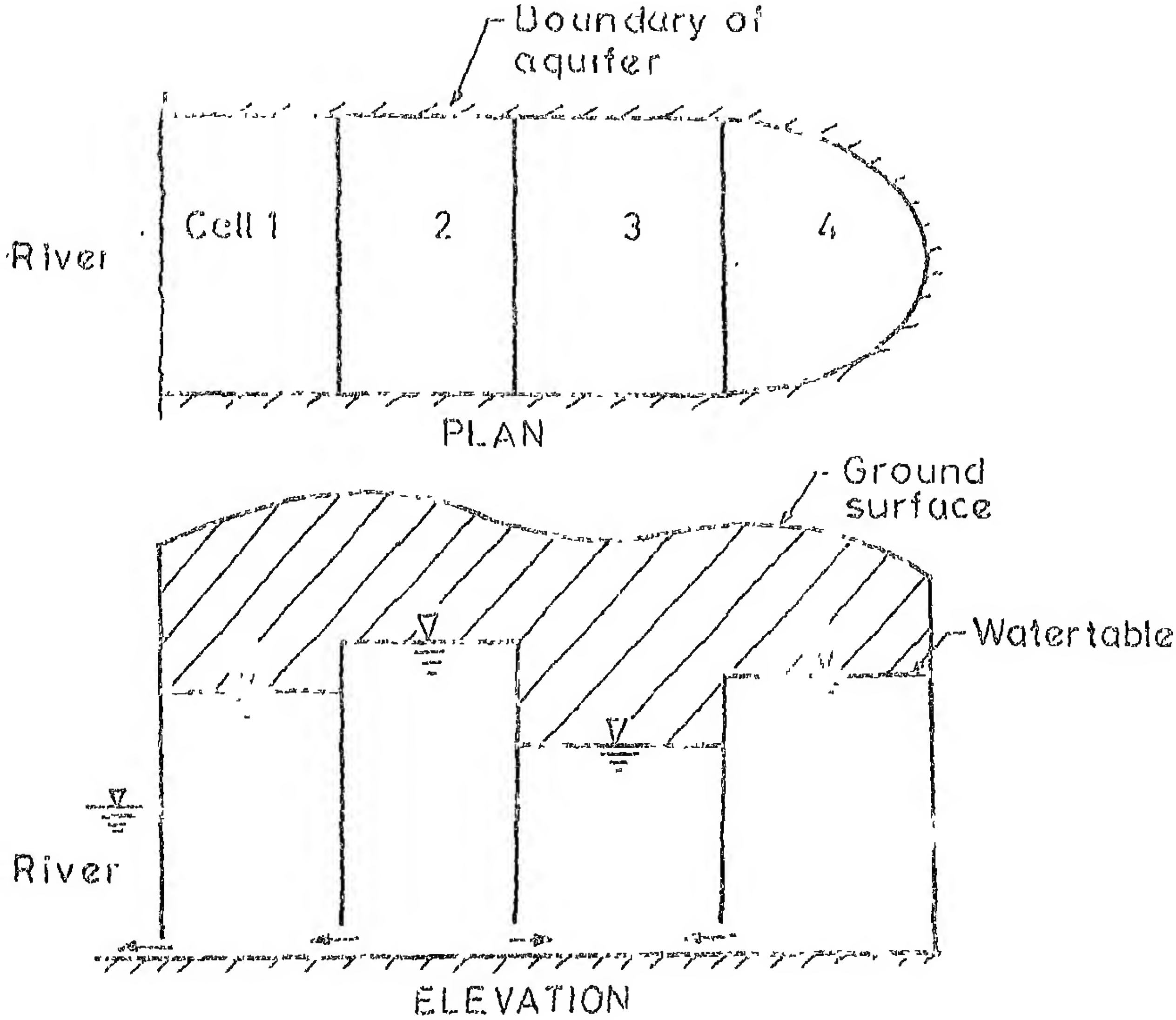


Fig 3·3 A four cell model of an aquifer

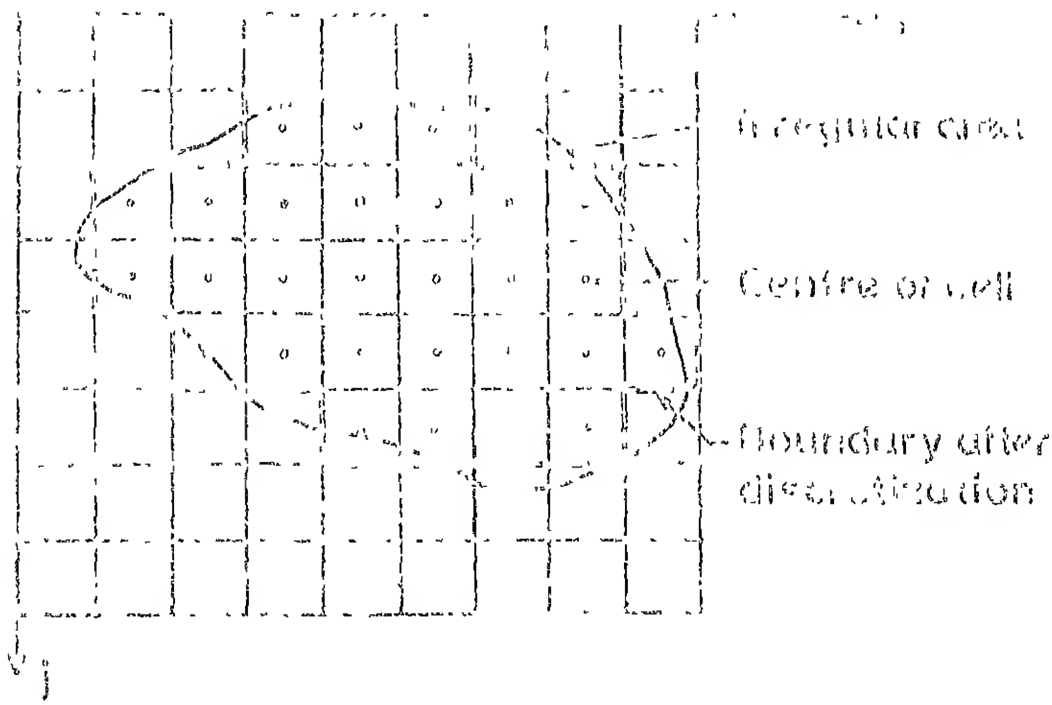


Fig.3.4 Discretization of irregular area
into cells

$S_{i,j} =$ Storativity of cell (i,j)

$r_{i,j}^{k+1/2}$ = Rate of recharge and per unit area of cell (i,j) at k^{th} time period

$p_{i,j}^{k+1/2}$ = Rate of pumping per unit area of cell (i,j) at k^{th} time period

$Q_x\big|_{i-1/2}$ = Discharge in direction x through the side at i-1/2 to the cell (i,j)

$h_{i,j}^{k+1/2}$ = Head at mid point between the instants k and k+1,

Now writing balance equation for cell (i,j) (Fig.3.5a,3.5b).

$$\Delta t \left\{ \left[Q_x\big|_{i-1/2} - Q_x\big|_{i+1/2} + Q_y\big|_{j+1/2} - Q_y\big|_{j-1/2} \right] + \Delta x_i \Delta y_j [r_{i,j}^t - p_{i,j}^t] \right\} = S_{i,j} \Delta x_i \Delta y_j [h_{i,j}^{t+\Delta t} - h_{i,j}^t] \tag{3.5}$$

By Darcy law Q = TWI

Put the values of Q_x, Q_y in Equation (3.5)

$$T_{i-1/2,j} \Delta y_j \frac{h_{i-1,j}^t - h_{i,j}^t}{[\Delta x_i + \Delta x_{i-1}]/2} + \frac{T_{i+1/2} \Delta y_j (h_{i+1,j}^t - h_{i,j}^t)}{(\Delta x_i + \Delta x_{i+1})/2}$$

$$+ T_{i,j+1/2} \Delta x_i \frac{(h_{i,j+1}^t - h_{i,j}^t)}{(\Delta y_j + \Delta y_{j+1})/2} + T_{i,j-1/2} \Delta x_i \frac{(h_{i,j-1}^t - h_{i,j}^t)}{(\Delta y_j + \Delta y_{j-1})/2}$$

$$+ (r_{i,j}^t - p_{i,j}^t) \Delta x_i \Delta y_j = S_{i,j} \frac{\Delta x_i \Delta y_j}{\Delta t} [h_{i,j}^{t+\Delta t} - h_{i,j}^t]$$

Taking the transmissivities at centres

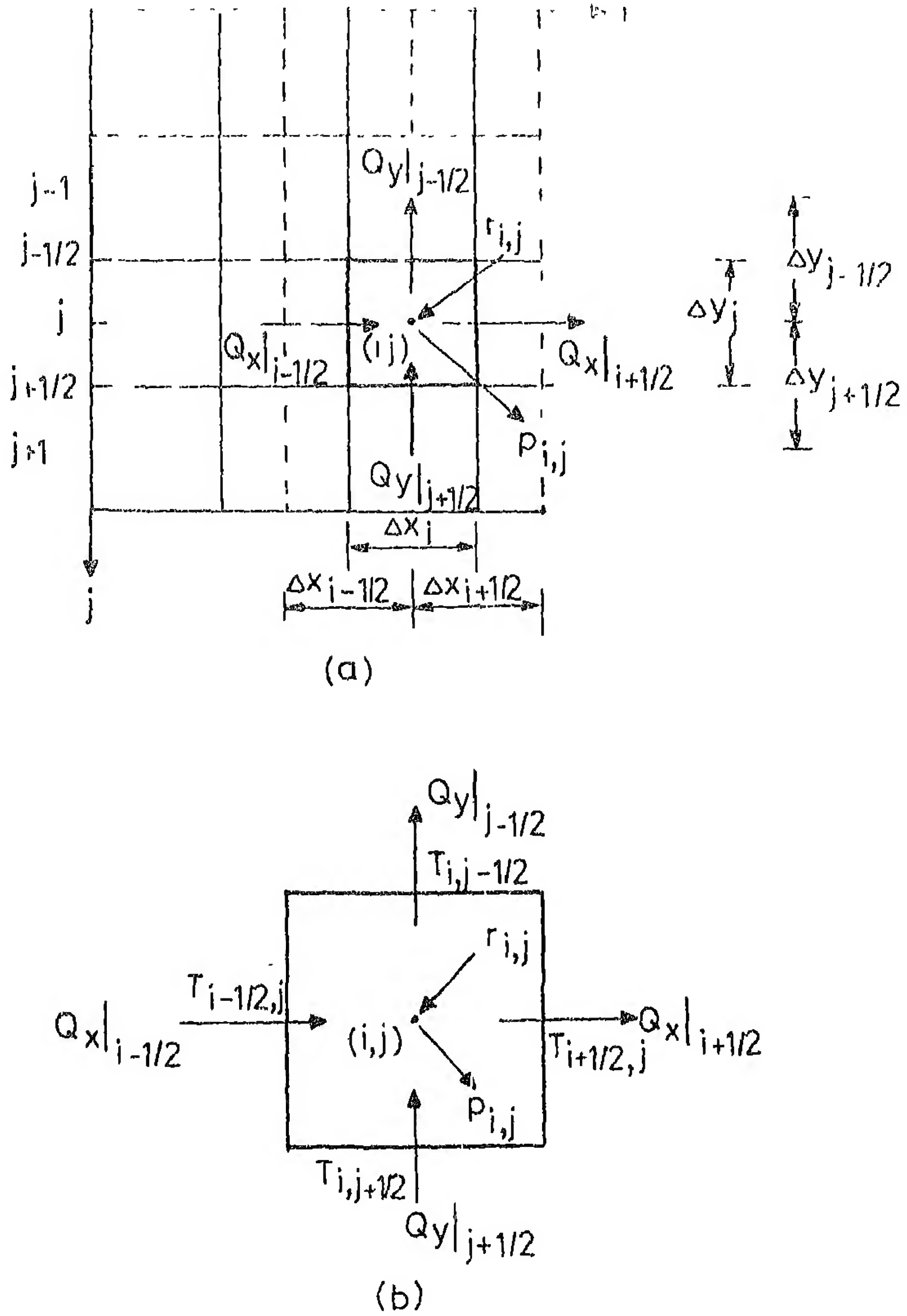


Fig. 3·5 Flow passing through a cell (I,J) in a multicell model

$$T_{i-1/2,j} = \frac{T_{i-1,j} + T_{i,j}}{2}$$

We get the balance equation

$$T_{i-1,j}\Delta y_j \frac{(h^t_{i-1,j} - h^t_{i,j})}{\Delta x_i + \Delta x_{i-1}} + T_{i+1,j}\Delta y_j \frac{(h^t_{i+1,j} - h^t_{i,j})}{(\Delta x_i + \Delta x_{i+1})}$$

$$+ T_{i,j+1}\Delta x_i \frac{(h^t_{i,j+1} - h^t_{i,j})}{(\Delta y_j + \Delta y_{j+1})} + T_{i,j-1}\Delta x_i \frac{(h^t_{i,j-1} - h^t_{i,j})}{(\Delta y_j + \Delta y_{j-1})}$$

$$+ T_{i,j}\Delta y_j \frac{(h^t_{i-1,j} - h^t_{i,j})}{\Delta x_i + \Delta x_{i-1}} + T_{i,j}\Delta y_j \frac{(h^t_{i+1,j} - h^t_{i,j})}{(\Delta x_i + \Delta x_{i+1})}$$

$$+ T_{i,j}\Delta x_i \frac{(h^t_{i,j+1} - h^t_{i,j})}{(\Delta y_j + \Delta y_{j+1})} + T_{i,j}\Delta x_i \frac{(h^t_{i,j-1} - h^t_{i,j})}{(\Delta y_j + \Delta y_{j-1})}$$

$$- S_{i,j}\frac{\Delta x_i \Delta y_j}{\Delta t}\left[h^{t+\Delta t}_{i,j} - h^t_{ij}\right] = (p^t_{i,j} - r^t_{i,j})\Delta x_i \Delta y_j$$

Putting

$$\Delta y_j + \Delta y_{j+1} = 2\Delta y_{j+1/2}$$

$$\Delta y_j + \Delta y_{j-1} = 2\Delta y_{j-1/2}$$

$$\Delta x_i + \Delta x_{i-1} = 2\Delta x_{i-1/2}$$

$$\Delta x_i + \Delta x_{i+1} = 2\Delta x_{i+1/2}$$

The equation can be written after dividing through by $\Delta x_i \Delta y_j$

$$\frac{(h^t_{i,j-1} - h^t_{i,j})}{2\Delta y_j \Delta y_{j-1/2}} T_{i,j-1} + \frac{(h^t_{i-1,j} - h^t_{i,j})}{2\Delta x_i \Delta x_{i-1/2}} T_{i-1,j}$$

$$+ \left[\frac{h^t_{i,j-1} - h^t_{i,j}}{2\Delta y_j \, \Delta y_{j-1/2}} + \frac{h^t_{i-1,j} - h^t_{i,j}}{2\Delta x_i \Delta x_{i-1/2}} + \frac{h^t_{i+1,j} - h^t_{i,j}}{2\Delta x_i \Delta x_{i+1/2}} \right.$$

$$\left. + \frac{h^t_{i,j+1} - h^t_{i,j}}{2\Delta y_j \Delta y_{j+1/2}} \right] . T_{i,j}$$

$$+ \frac{(h^t_{i+1,j} - h^t_{i,j})}{2\Delta x_i \, \Delta x_{i+1/2}} T_{i+1,j} + \frac{h^t_{i,j+1} - h^t_{i,j}}{2\Delta y_j \Delta y_{j+1/2}} T_{i,j+1}$$

$$- \frac{h^{t+\Delta t}_{i,j} - h^t_{i,j}}{\Delta t} S_{i,j} = (p^t_{i,j} - r^t_{i,j}) \qquad (3.6)$$

Another scheme is based on the CrankNicholsen method, where the spatial gradients are computed at the mid point of the time interval.

At the beginning, head $= h^t_{i,j}$

At the end of time interval, head $= h^{t+\Delta t}_{i,j}$

At the mid point of time interval, head $= h^{t+\Delta t/2}_{i,j}$

Suffix k represents the time instant. Then at mid point

$$h^{k+1/2}_{i,j} = \frac{h^k_{i,j} + h^{k+1}_{i,j}}{2}$$

The equation (3.6) can now be written as

$$a^{k+\frac{1}{2}}_{i,j} T_{i,j-1} + b^{k+\frac{1}{2}}_{i,j} T_{i-1,j} + c^{k+\frac{1}{2}}_{i,j} T_{i,j} + d^{k+\frac{1}{2}}_{i,j} T_{i+1,j}$$

$$+ e^{k+1/2}_{i,j} T_{i,j+1} + f^{k+1/2}_{i,j} S_{i,j} = p^{k+1/2}_{i,j} - r^{k+1/2}_{i,j}$$

$$\text{or } \sum_{m=1}^{5} g_m T_m + f^{k+1/2}_{i,j} S_{i,j} = p^{k+1/2}_{i,j} - r^{k+1/2}_{i,j} \qquad (3.7)$$

a,b,c,d,e,f are known as LHS coefficient of balance equation

where,

$$a_{i,j}^{k+1/2} = \frac{h_{i,j-1}^{k+1/2} - h_{i,j}^{k+1/2}}{2\,\Delta y_j\ \Delta y_{j-1/2}} \qquad (3.7a)$$

$$b_{i,j}^{k+1/2} = \frac{h_{i-1,j}^{k+1/2} - h_{i,j}^{k+1/2}}{2\,\Delta x_i\ \Delta x_{i-1/2}} \qquad (3.7b)$$

$$c_{i,j}^{k+1/2} = \frac{h_{i,j-1}^{k+1/2} - h_{i,j}^{k+1/2}}{2\Delta y_j\ \Delta y_{j-1/2}} + \frac{h_{i-1,j}^{k+1/2} - h_{i,j}^{k+1/2}}{2\,\Delta x_i \Delta x_{i-1/2}}$$

$$+ \frac{h_{i+1,j}^{k+1/2} - h_{i,j}^{k+1/2}}{2\,\Delta x_i\, \Delta x_{i+1/2}} + \frac{h_{i,j+1}^{k+1/2} - h_{i,j}^{k+1/2}}{2\,\Delta y_j\ \Delta y_{j+1/2}} \qquad (3.7c)$$

$$d_{i,j}^{k+1/2} = \frac{h_{i+i,j}^{k+1/2} - h_{i,j}^{k+1/2}}{2\,\Delta x_i\ \ \Delta x_{i+1/2}} \qquad (3.7d)$$

$$e_{i,j}^{k+1/2} = \frac{h_{i,j+1}^{k+1/2} - h_{i,j}^{k+1/2}}{2\,\Delta y_j\ \Delta y_{j+1/2}} \qquad (3.7e)$$

$$f_{i,j}^{k+1/2} = \frac{h_{i,j}^{k} - h_{i,j}^{k+1}}{\Delta t} \qquad (3.7f)$$

The coefficients of the balance equation, a,b,c,d,e,f are calculated for each cell and balance equation for each cell can be written in the form of finite difference equation for each time period.

3.4 SOLUTION METHOD:

For true values of T and S, the equation (3.7) must be satisfied and values of its LHS and RHS will be equal.

If number of cells	= $\not<$	C
Time steps	=	N
The number of equation	=	NC
and unknown (T and S)	= $\not>$	2C

[Kisiel and Duckstein ,1972].

The system is called,

Under determined	if	$NC < 2C$
Determined	if	$NC = 2C$
Over determined	if	$NC > 2C$

If the set of 2C equations is solved and the model is a true representation of the aquifer, the results obtained will be the true values of S and T. But rarely a model is a true representation of the practical conditions so the values (from model) are not true values. This discripency introduces certain noise in each equation. Each equation becomes an approximation . Since all equations are approximations, our goal will be to find the value which is nearest to the true solution i.e. here the problem becomes a problem of finding an optimal set of values which will satisfy the equations most closely. This is done with the help of linear or quadratic programming. Certain criteria are fixed to find a close from solution.

As stated above result values and the actual values of the model are different. LHS values of balance equation (3.7) will not be equal to the RHS of the balance equation. For a better solution the deviation between these two sides of the equation must be minimum. The different criteria to get optimal solution are based upon this deviation.

The problem is reduced to LP or NLP problem with optimizing the objective function (linear or quadratic) subject to given linear constraints.

3.5 CRITERIA FOR OPTIMIZATION:

The following five criteria will be considered. The first four criterions use linear programming whereas the fifth criterion uses the quadratic programming.

3.5.1 Criterion A:

This criterion minimizes the maximum absolute deviation between two sides of equation (3.7) for all the cells for all time intervals.

Let the value of maximum absolute deviation be X. Then the absolute value of any other deviation occurring for any time period for any cell will be less than or equal to X. Therefore objective function is

$$\text{Min} \quad F = X$$

$$\text{or} \quad \text{Max} \quad Z = -X \tag{3.8}$$

Subject to

$$\left| \sum_{m=1}^{5} g_m T_m + f_{i,j}^{k+1/2} S_{i,j} - (p_{i,j}^{k+1/2} - r_{i,j}^{k+1/2}) \right| \leq X$$

This constraints is equivalent to two constraints.

$$\sum_{m=1}^{5} g_m T_m + f_{i,j}^{k+1/2} S_{i,j} - X \leq p_{i,j}^{k+1/2} - r_{i,j}^{k+1/2}$$

$$- \sum_{m=1}^{5} g_m T_m - f_{i,j}^{k+1/2} S_{i,j} - X \leq -(p_{i,j}^{k+1/2} - r_{i,j}^{k+1/2}) \tag{3.9}$$

$$T_{i,j}\ ,\ S_{i,j},\ X \geq 0 \quad \forall\ i,j,k\ .$$

Number of decision variables = 2C + 1

Number of constraints = 2NC

where N = number of time periods

C = number of cells.

3.5.2 Criterion B:

This criterion minimizes the sum of the absolute values of maximum deviation for all time intervals. For each time interval there are C balance equations. Let $X^{k+1/2}$ be maximum absolute deviation corresponding to $(k+1/2)^{th}$ time interval. Then all other deviations for that time interval will be less then or equal to $X^{k+1/2}$

$$\left| \sum_{m=1}^{5} g_m T_m + f_{i,j}^{k+1/2} S_{i,j} - (p_{i,j}^{k+1/2} - r_{i,j}^{k+1/2}) \right| \leq X^{k+1/2} \quad \forall\ i,j$$

Number of such deviations will be equal to the number of time periods. The LP problem is formulated as

$$\text{Min } F = \sum_{k=0}^{N-1} X^{k+1/2}$$

$$\text{or} \qquad \text{Max } Z = - \sum_{k=0}^{N-1} X^{k+1/2} \tag{3.10}$$

Subject to

$$\sum_{m=1}^{5} g_m T_m + f_{i,j}^{k+1/2} S_{i,j} - X_{i,j}^{k+1/2} \leq p_{i,j}^{k+1/2} - r_{i,j}^{k+1/2} \tag{3.11}$$

$$- \sum_{m=1}^{5} g_m T_m - f_{i,j}^{k+1/2} S_{i,j} - X_{i,j}^{k+1/2} \leq -(p_{i,j}^{k+1/2} - r_{i,j}^{k+1/2})$$

$$X_{ij}^{k+1/2}, T_{i,j}, S_{i,j} \geq 0 \quad \text{for all } i,j,k.$$

Number of decision variables = 2C + N

Number of constraints = 2NC.

3.5.3 Criterion C;

This criterion minimizes the sum of the absolute values of the maximum deviations for all cells. Number of maximum deviations will be C as number of cells is C. Let the maximum absolute deviation for cell (i,j) be $X_{i,j}$. Then all other deviations at this cell will having lesser or equal values to this.

$$\left| \sum_{m=1}^{5} g_m T_m + f_{i,j}^{k+1/2} S_{i,j} - p_{i,j}^{k+1/2} + r_{i,j}^{k+1/2} \right| \leq X_{i,j} \quad \forall k \tag{3.12}$$

The problem now reduces to an LP problem which states, to find $S_{i,j}$ and $T_{i,j}$ with,

Subject to

$$\sum_{m=1}^{5} g_m T_m + f_{i,j}^{k+1/2} S_{i,j} - X_{i,j}^{k+1/2} \leq p_{i,j}^{k+1/2} - r_{i,j}^{k+1/2}$$

$$\sum_{m=1}^{5} g_m T_m - f_{i,j}^{k+1/2} S_{i,j} - X_{i,j}^{k+1/2} \leq -(p_{i,j}^{k+1/2} - r_{i,j}^{k+1/2}) \qquad (3.15)$$

$$S_{i,j}, T_{i,j}, X_{i,j}^{k+1/2} \geq 0 \quad \forall\ i,j,k.$$

Number of decision variables = C(2+N)

Number of constraints = 2NC.

In above four criterion the objective function was linear in next criterion, quadratic form of the objective function is used.

3.5.5 Criterion E:

This criterion minimizes the sum of the squares of the deviations for all cells for all time periods.

Let $X_{i,j}^{k+1/2}$ be the deviation for cell i,j for time period (k+1/2).

Objective function becomes

$$\text{Min } F = \sum_i \sum_j \sum_k (X_{i,j}^{k+1/2})^2 \qquad (3.16)$$

Subject to

$$\sum_{m=1}^{5} g_m T_m + f_{i,j}^{k+1/2} S_{i,j}^{\cancel{k+1/2}} = p_{i,j}^{k+1/2} - r_{i,j}^{k+1/2} \qquad (3.17)$$

$$\sum_{m=1}^{5} g_m T_m + f_{i,j}^{k+1/2} S_{i,j} - X_{i,j}^{k+1/2} \geq p_{i,j}^{k+1/2} - r_{i,j}^{k+1/2}$$

$$\text{Min } F = \sum_i \sum_j X_{i,j}$$

or $$\text{Max } Z = - \sum_i \sum_j X_{i,j}$$

Subject to

$$\sum_{m=1}^{5} g_m T_m + f_{i,j}^{k+1/2} S_{i,j} - X_{i,j} \leq p_{i,j}^{k+1/2} - r_{i,j}^{k+1/2}$$

$$- \sum_{m=1}^{5} g_m T_m - f_{i,j}^{k+1/2} S_{i,j} - X_{i,j} \leq -(p_{i,j}^{k+1/2} - r_{i,j}^{k+1/2})$$

(3.13)

$$T_{i,j},\ S_{i,j},\ X_{i,j} \geq 0 \qquad \forall\ i,j$$

Number of decision variables = 3C.

Number of constraints = 2CN

3.5.4 Criterion D:

This criterion minimizes the sum of the absolute values of all deviations for all cells for all time intervals.

Let $X_{i,j}^{k+1/2}$ be the deviation for cell (i,j) for time period $(k+1/2)$

Then,

$$\left| \sum_{m=1}^{5} g_m T_m + f_{i,j}^{k+1/2} S_{i,j} - p_{i,j}^{k+1/2} + r_{i,j}^{k+1/2} \right| \leq X_{i,j}^{k+1/2}$$

(3.14)

The problem reduces to the LP problem as

$$\text{Min } F = \sum_i \sum_j \sum_k X_{i,j}^{k+1/2} \qquad \forall\ j,j,k$$

or $$\text{Max } Z = - \sum_i \sum_j \sum_k X_{i,j}^{k+1/2}$$

$$- \sum_{m=1}^{5} g_m T_m - f_{i,j}^{k+1/2} S_{i,j} + X_{i,j}^{k+1/2} \geq -(p_{i,j}^{k+1/2} - r_{i,j}^{k+1/2})$$

(3.18)

$$T_{i,j}, S_{i,j} \geq 0 \quad \forall\ i,j,k$$

Number of decision variables = (2+N) C

Number of constraints = 2 NC.

To take into account the negative values of X, a large positive number Y' is added to X, such that the sum is always positive. Let

$$Y = X + Y'$$

So $\quad X = Y - Y'$

The problem can be written as

$$\text{Min } F = \sum_i \sum_j \sum_k (Y_{i,j}^{k+1/2} - Y')^2$$

Subject to

$$\sum_{m=1}^{5} g_m T_m + f_{i,j}^{k+1/2} S_{i,j} - Y_{i,j}^{k+1/2} \geq p_{i,j}^{k+1/2} - r_{i,j}^{k+1/2} - y'$$

$$- \sum_{m=1}^{5} g_m T_m - f_{i,j}^{k+1/2} S_{i,j} + Y_{i,j}^{k+1/2} \geq -(p_{i,j}^{k+1/2} - r_{i,j}^{k+1/2}) + Y'$$

(3.19)

$$T_{i,j}, S_{i,j}, Y_{i,j}^{k+1/2} \geq 0, \quad \forall\ i,j,k.$$

The problem can also be solved by putting the value of $X_{i,j}^{k+1/2}$ from equation (3.17) in Equation (3.16). Then the quadratic problem can be stated as

$$\text{Min } F = \sum_i \sum_j \sum_k \Big[\sum_{m=1}^{5} g_m T_m + f_{i,j}^{k+1/2} S_{i,j} - (p_{i,j}^{k+1/2} - r_{i,j}^{k+1/2}) \Big]^2$$

(3.20)

Subject to $T_{i,j} \geq 0$

$S_{i,j} \geq 0 \quad , \quad \forall\ i,j,k$.

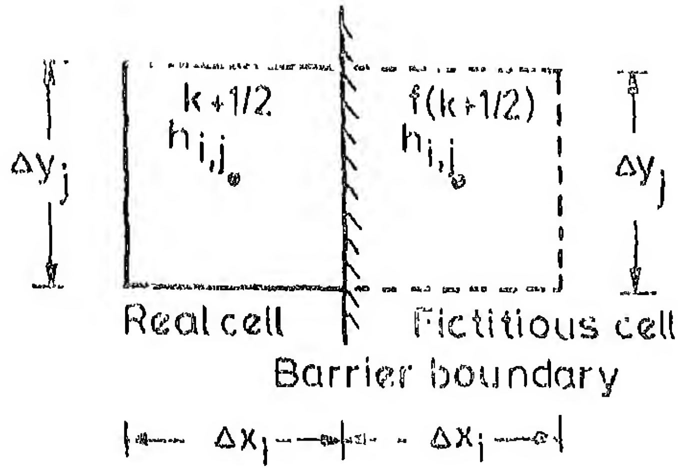
3.6 REPRESENTATION OF AQUIFER BOUNDARIES:

The boundary condition of aquifer may be no flow boundary, constant head boundary or varying head boundary. All these conditions are taken care of by the values of the coefficients of the balance equation. This is done in following manner:

Consider a cell (i,j) at the boundary (Figs. 3.6a and 3.6b) . The flow to cell (i,j) will be governed by the transmissivities of all cells adjoining this cell. To incorporate the flow from boundary consider a fictitious cell, beyond the boundary. The head in the fictitious cell is $h^{f,(k+1/2)}_{i,j}$ with its transmissivity as $T^{f}_{i,j}$.

In case the boundary is a barrier boundary there will be no flow across the boundary so the head $h^{f,(k+1/2)}_{i,j}$ is taken same as $h^{k+1/2}_{i,j}$. The balance equation of the cell (i,j) will not have $T^{f}_{i,j}$.

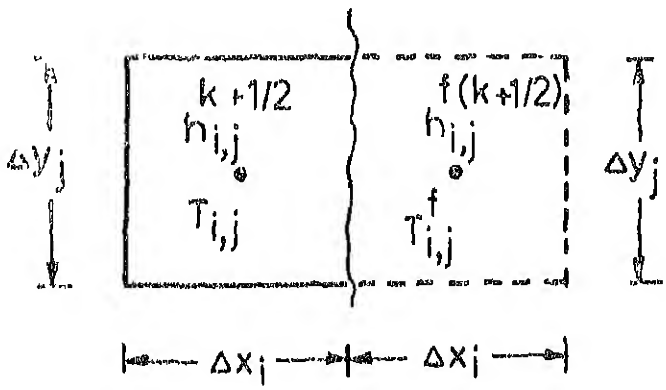
In case the boundary is a pervious boundary, the value of $h^{f,(k+1/2)}_{i,j}$ is kept equal to the actual head in fictitious cell. This may be constant head or varying head with time. Now the corresponding coefficient is calculated using these head values and the balance equation for cell (i,j) is written. In this case one extra transmissivity value has to



Barrier boundary

$$h_{ij}^{k+1/2} = h_{ij}^{f,(k+1/2)}$$

(a)



Pervious boundary

$h_{i,j}^{f(k+1/2)}$ = actual head in fictitious cell

(b)

Fig.3·6 Representation of aquifer boundaries

be calculated for each fictitious cells. Hence the number of parameters to be found will become equal to twice the number of cells plus the number of fictitious cells.

3.7 APPLICATION OF WEIGHTS IN OBJECTIVE FUNCTION:

The data used for the problem consists of the dimensions of cells, heads and the pumpage or recharge (some times known as source/sink values). The values of heads are obtained from piezometric wells located in the area. It is evident that the cell which has more number of piezometers located in it, will be having more reliable data than others which have less number, or the data in cells far from the piezometric wells will be less reliable, as compared to those which are comparatively nearer to the piezometers. Using this principle certain terms known as 'reliability index' have been calculated to represent the relative reliability of data available in particular cells. These terms were assigned to the cost coefficient in the objective function to have more realistic results.

The procedure to find these indices is as follows: Draw the position of the piezometric wells in the case study area. The area represented by each piezometer P is found by drawing Thiessen Polygons (Fig. 3.7). Now find the total area represented by piezometer P_1 and the cells contributing area to this piezometer. The data of the cell having its area inside area (partially or fully) represented by any piezometer will be less reliable if area represented by piezometer is large,

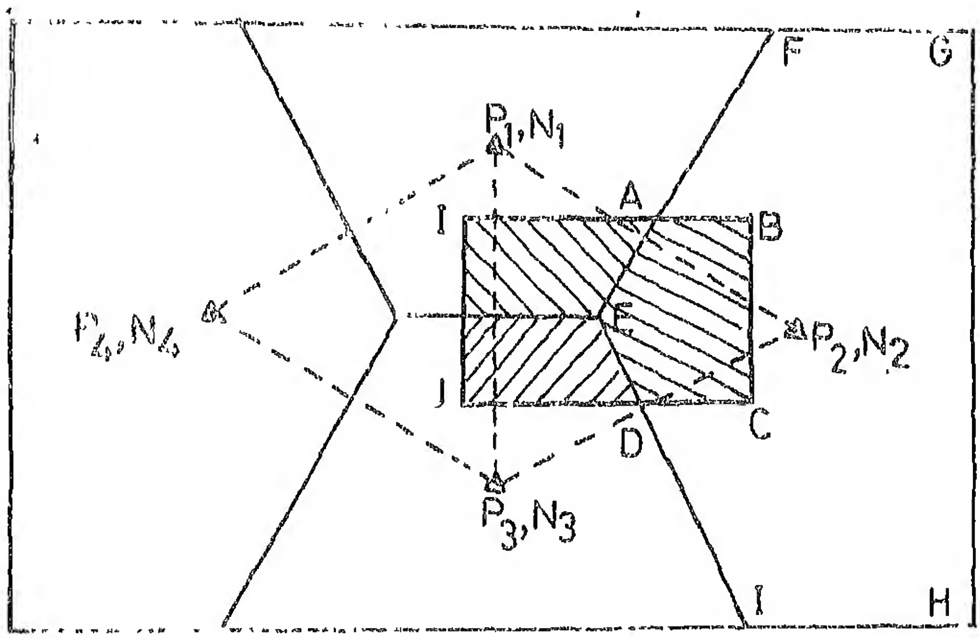
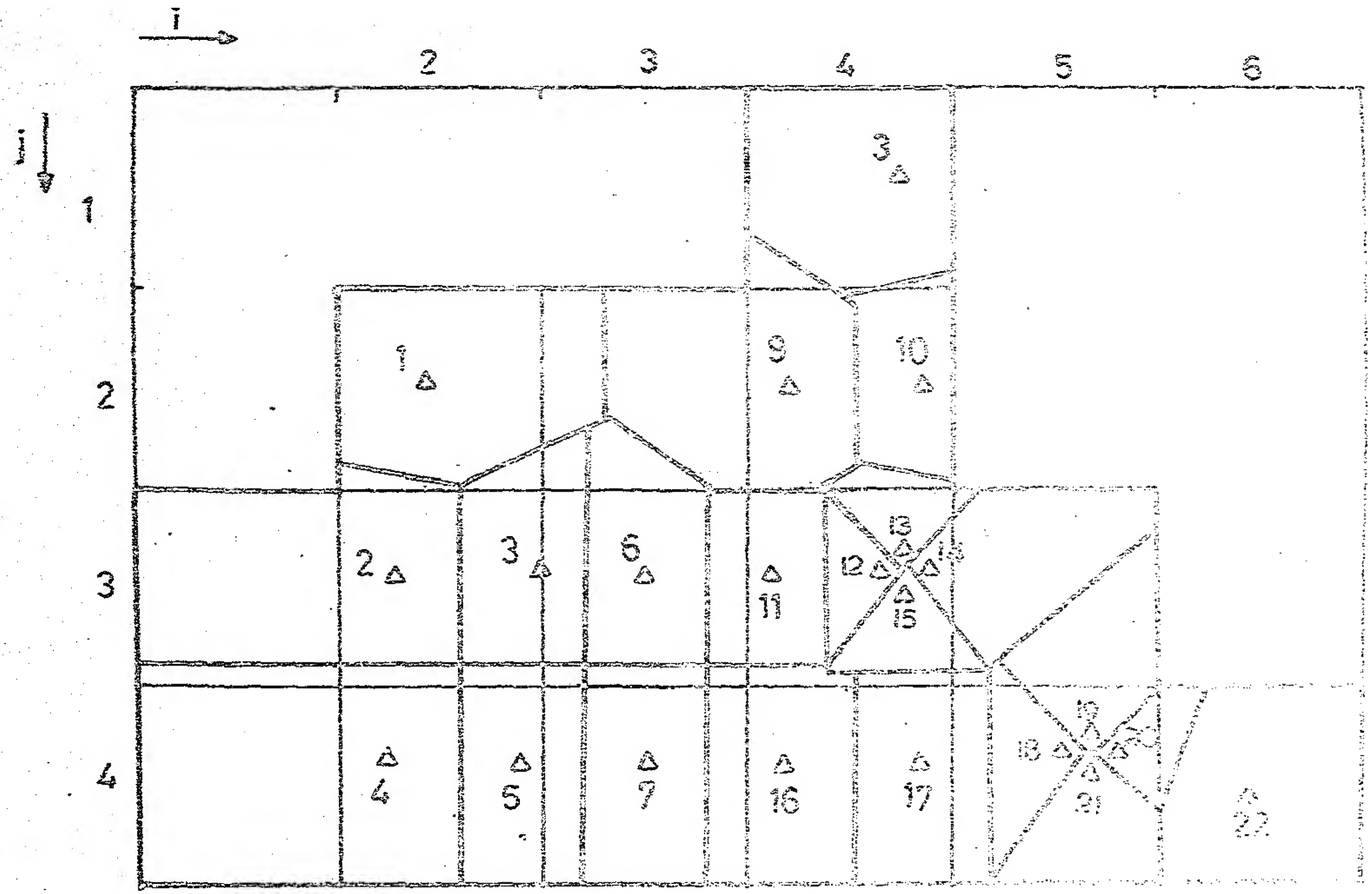


Fig.3·7 Thiessen polygons showing area represented by piezometers of an aquifer



Δ-Shows the position of piezometer

Fig.3·8 Thiessen polygons showing the area represented by piezometers in the aquifer

i.e. the reliability of data is inversely proportional to area of polygon. A number N, which is representative of a relative reliability of the data for area represented by the piezometer P_i is obtained by dividing total area of basin by the area of corresponding polygon. Now relative reliability of the polygons is known as number N. To find the reliability of data in a cell the following procedure is adopted.

Let P_1, be the piezometers (P_1, P_2, P_3 in Fig. 3.7). The thessen polygon are drawn. Reliability index R of cell,

$$R_{i,j} = \sum \frac{\text{Area of polygon } P_i \text{ covered by cell}}{\text{Total area of the polygon } P_i} \times N_i \quad \forall \ i$$

$$= \frac{\text{Area ABCDE}}{\text{Area EFGHI}} \times N_2 + \ldots\ldots \tag{3.18}$$

Weightage of cell (i,j) can now be calculated as,

$$w_{i,j} = \frac{R_{i,j}}{\Sigma R_{i,j}} \tag{3.19}$$

CHAPTER - 4

APPLICATION

In this chapter the application of theory presented in Chapter 3 is discussed. Different shapes of aquifers and different boundary conditions are selected for the study. The data used are based on certain realistic situations.

4.1 DIFFERENT AQUIFER CONDITIONS:

4.1.1 Rectangular Aquifer:

A six cell model for the aquifer it considered, (Fig. 4.1a,b,c) . The boundary conditions chosen are:

(i) all the sides are barrier boundaries,

(ii) two sides have varying head and two sides are barrier boundaries,

(iii) two sides have constant head and the other two sides are barrier boundaries.

4.1.2 A Stepped Aquifer:

A fifteen cell model for the aquifer is considered with all sides as barrier boundaries(Fig. 4.2).

4.2 DATA USED:

4.2.1 Time Periods:

The heads data are assumed to be available for three time periods. The duration for these periods are 125,132 and

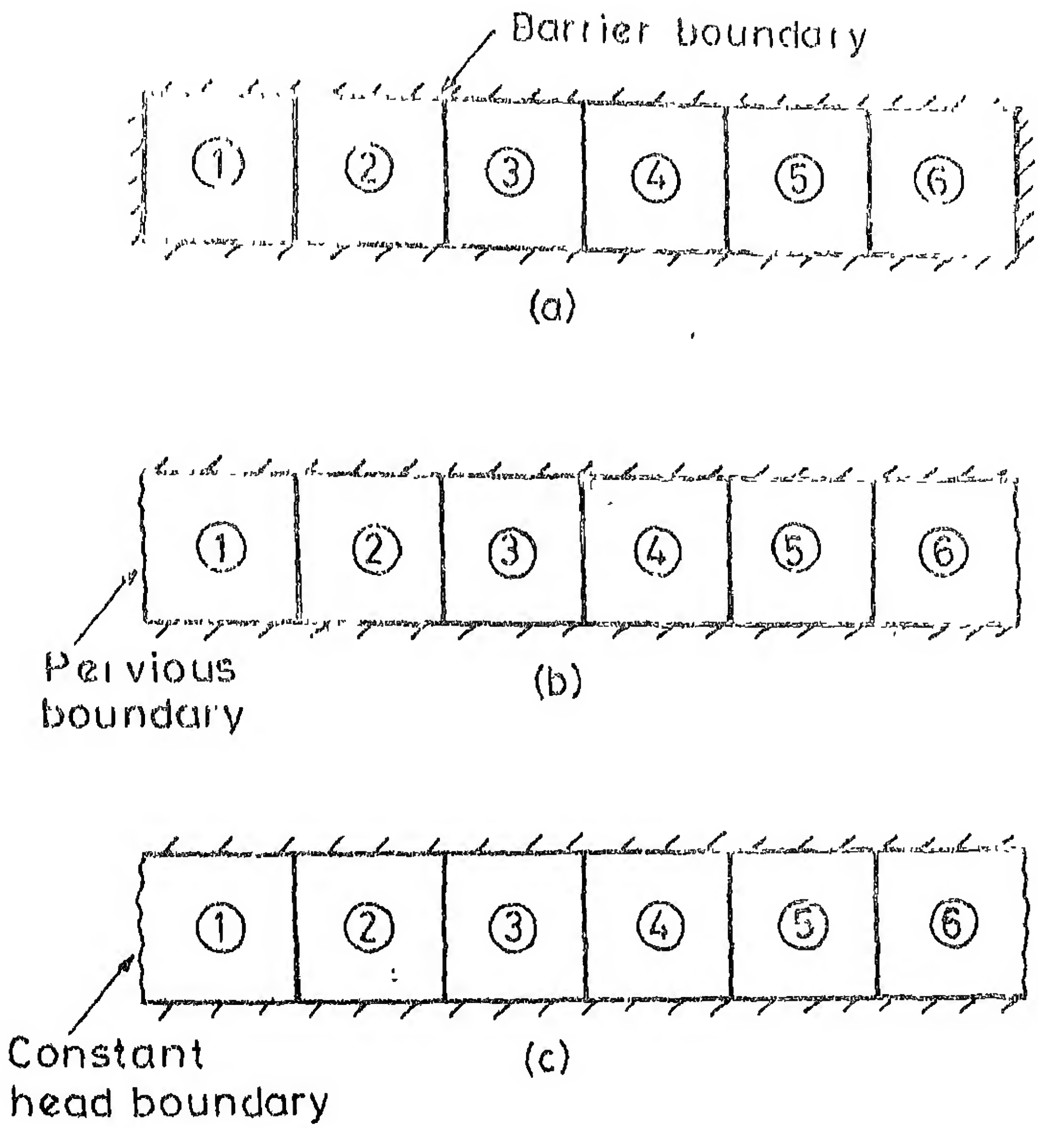


Fig. 4.1 Six cell model of the aquifer with different boundary conditions

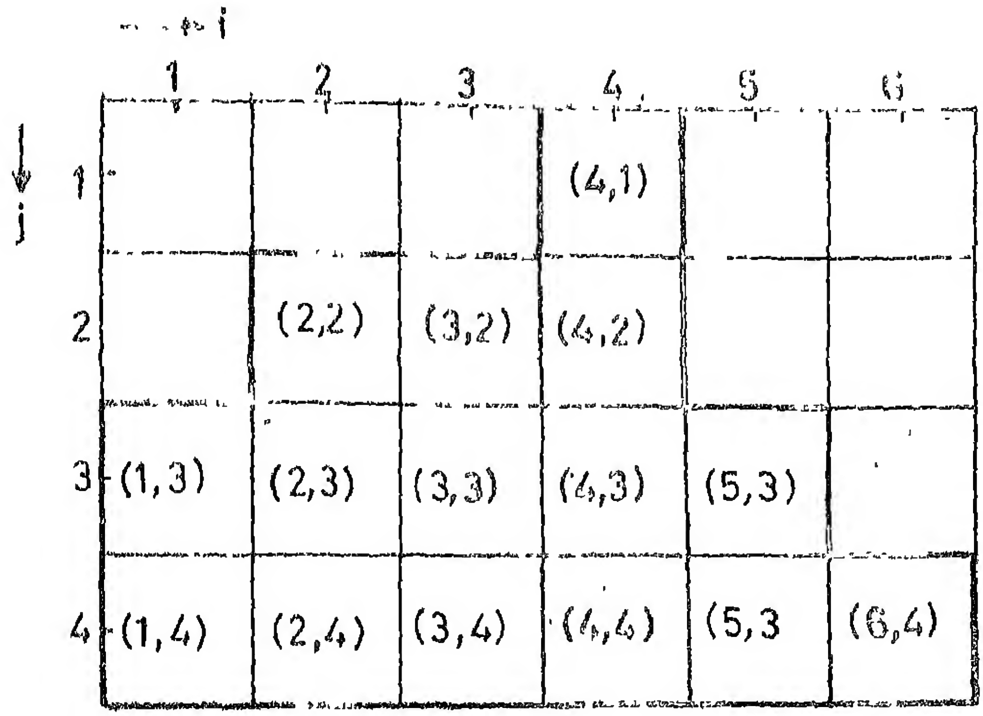


Fig.4·2 A stepped aquifer with barrier boundaries

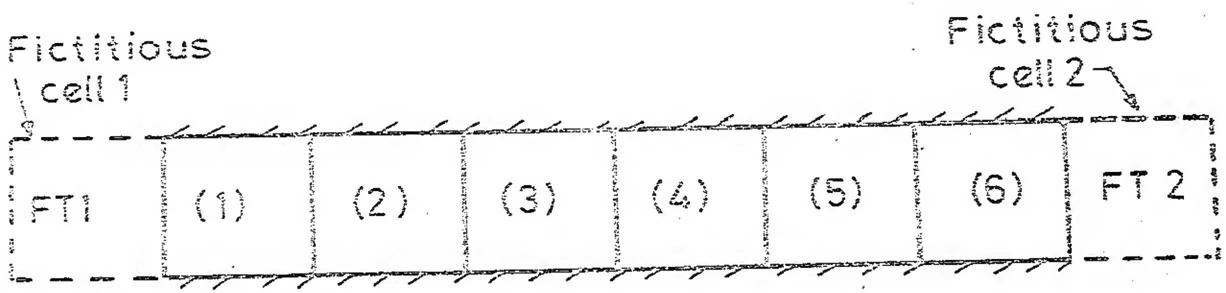


Fig.4.3 The pervious boundary replaced by fictitious cell FT1, FT2

for the study are shown in Table 4.3a.

4.2.5 Left Hand Side Coefficients:

The coefficients a,b,c,d,e,f of the equation (3.7) are termed LHS coefficients. The definition sketch of these coefficients has been shown in Fig. 4.4. The values of these coefficients are calculated by putting values of $h^{k}_{i,j}$, $h^{k+1}_{i,j}$, $h^{k+1/2}_{i,j}$ and Δt in equations 3.7(a) to 3.7(f). The values of these coefficients are assigned to cells.

4.2.6 RHS Coefficients of the Balance Equation:

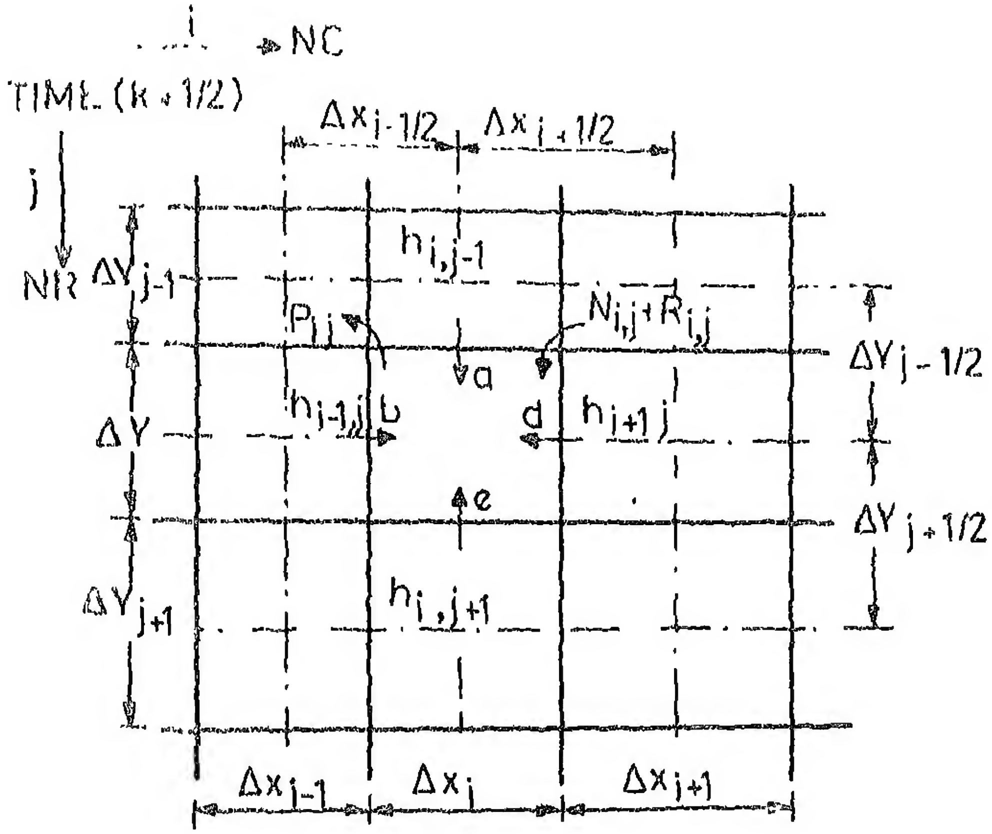
The RHS of the balance equation comprises of $p^{k+1/2}_{i,j}$ - $r^{k+1/2}_{i,j}$ and these are calculated by putting the values of $p^{k+1/2}_{i,j}$ and $r^{k+1/2}_{i,j}$. The net values assigned to particular cells for particulat time period.

4.2.7 Storage Coefficient and Transmissivity Values:

To apply the modified optimization, estimates of T and S i.e. the approximate value of T and S are required in different cells. For the cells where these values are not available reasonable values are assumed and shown in Table 4.4.

4.2.8 Weights for Objective Function:

The weights which have to be assigned to various cells are calculated from Fig. 3.8 Values of reliability indices and weightage factors are given in Table 4.5. The weights



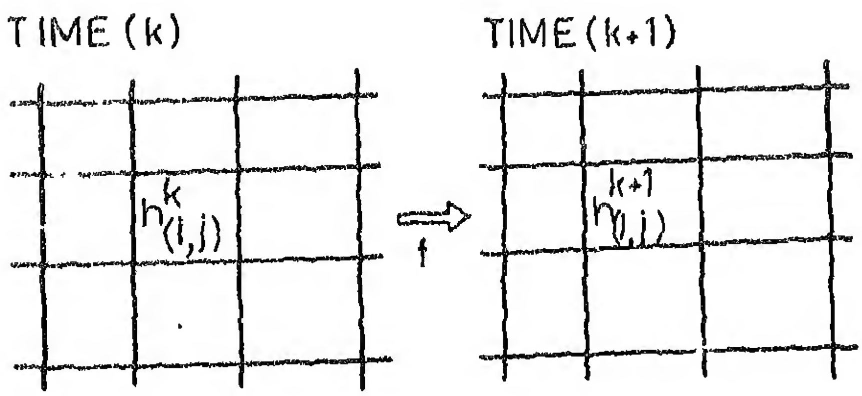


Fig.4·4 Coefficients a,b,d,e,f corresponding to the cell (i,j)-definition sketch

are applied to the decision variables in the objective function. The objective function can be written as

$$\text{Min } F = \sum_i \sum_j \sum_k w_{i,j} (X_{i,j}^{k+1/2})^2 \qquad (4.1)$$

4.2.9 Modified Objective Function:

The modified objective function is written from equation (2.14) as

$$F_{mod} = \sum_i \sum_j \sum_k (X_{i,j}^{k+1/2})^2 + d \left[\sum_i \sum_j \left(1 - \frac{T_{i,j}}{T_{i,j}^e}\right)^2 + \sum_i \sum_j \left(1 - \frac{S_{i,j}}{S_{i,j}^e}\right)^2 \right]$$

(4.2)

The expected values of $T_{i,j}^e$ and $S_{i,j}^e$ are put in the expression and the coefficients of terms involving squares and linear terms of $S_{i,j}$ and $T_{i,j}$ are calculated and given in Table 4.6 .

CHAPTER - 5

RESULTS DISCUSSION AND CONSLUSIONS

Five different criteria were presented in Chapter 3. The results of applying these criteria to different aquifer situations are given in Table 5.1.1 to 5.4.3.

5.1 OPTIMIZATION CRITERIA:

5.1.1 Criterion A minimizes the absolute value of the maximum deviation occuring in the balance equations. The criterion will give reasonable results as long as the absolute value of maximum deviation is very high compared to the other deviations. As only maximum deviation appears in the objective function. The linear programming problem is not concerned with the values of remaining deviations. However, if the remaining deviations have values marginally smaller than the maximum deviation the criterion will fail to give reasonable results.

5.1.2 Criterion B minimizes the sum of the absolute values of maximum deviation occuring for all time periods. This criterion will choose the maximum deviation occuring for each time period and will minimize the sum of their absolute values. This criterion is stated explictly time wise but not space wise in the objective function. The criterion is likely to give reasonable results only if the number of time period is high. If the problem has one time period criterion A and B become identical.

5.1.3 Criterion C minimizes the maximum absolute values for all cells. The criterion is explicit spacewise but not time wise. More the number of cells, better will be the results with this criterion. In the present study the number of cells is greater than number of time periods. So the results with this criterion are likely to be nearer to real values of the parameters. If the number of cells is one the criterion A and C will become identical in optimization.

5.1.4 Criterion D minimizes the sum of the absolute values of all deviations. The criterion is explicit time wise as well as space wise i.e. all are represented in the objective function for all time periods.

5.1.5 Criterion E minimizes the sum of the squares of the residuals occuring between LHS and RHS of all balance equations. In criterion D the absolute values of the deviations were considered. Some times criterion C and D do not have feasible solutions. For example when the number of cells were increased to 15 cells criteria C and D did not have feasible results. This problem was again encountered in the case of 6 cell model with constant head on two sides and barrier boundaries on the other two sides (5.3.1).

5.2 DISCUSSION OF RESULTS:

In deciding which criterion is better, the results are compared with field values, given in Table 4.4.

From this Table it is seen that for the 6 cell model T varies from 120 m^2/day to 170 m^2/day and S varies from 0.06 to 0.147. The results for three different boundary conditions were analyzed and discussed below.

5.2.1 6 Cell Model with Barrier Boundaries on all Sides:

The Table 5.1.1 shows that when criterion A is used, three values for T become zero and two values are high namely 268.75, 304.40 m^2/day. Criterion B also gives two values of T very high , 244.60 , 242.16 m^2/day. Criterion D which gives reasonable T values gives three S values as zero compared to two zero S values in the case of A , C and E. For criterion E one value of T is high, 237.50 m^2/day.

Thus, criterion C gives better results than the other criteria.

On comparing the results in Tables 5.1.1 and 5.1.2 it is observed that criterion C still gives the best results.

From Table 5.1.3, it is seen that the values are better at $d = 0.004$, when weights are used. However if these results are compared with those obtained from Table 5.1.1 than T_3, T_4, T_5 are nearer to the approximate values given in Table 5.1.3 but S values are better for S_1, S_4, S_6. Thus while no definite conclusion can be drawn as to which criterion is best, it appears that both criteria, criterion C and criterion E with modified optimization at $d = 0.004$, (using weights) are equally good for this case.

5.2.2 6 Cell Model Having Varying Head Boundaries on Two Sides andBarrier Boundaries on the Other Two Sides:

If results obtained in Table 5.2.1 are analyzed it is seen that criterion A gives non zero values to T_2, T_3, T_5 only , in which T_2 and T_5 (235.00, 201.44 m^2/day respectively) are high and T_3 (87.5 m^2/day) is low. Criterion B gives zero values for two cells for both parameters S and T. But T_2,T_5 (220 , 253.50 m^2/day) are higher than maximum value of T. Criterion D gives zero values of S to 3 cells compared to two zero values of S in criterion C. Also the values of T_5 (202.25 m^2/day) is very high. Criterion E assigns two zero values to T, and two zero values to S i.e. same as criterion C. But T_5 (243.54 m^2/day) is high. Therefore criterion C gives the best results where T_3,T_4,T_5 are very near to expected values and only two S values are zero.

Comparing the results of Table 5.2.2, for different values of d , it is seen that d = 0.06 gives a set of better results. Only one S value has zero value and the values of T are almost same for different d.

If the results obtained from Table 5.2.3 are analyzed it is seen that if weights are used the criterion E with d = 0 has better results than those with other values of d. As for other values of d deviation in S values from their expected values is much larger. T is almost same. However,

if this set is compared with the results obtained with criterion C it is seen that overall the criterion C gives the best results for this case.

5.2.3 6 Cell Model with Constant Head Boundaries on Two Sides and Barrier Boundaries on Remaining Sides:

Table 5.3.1 shows that criterion A gives T_2 a high value (342.34 m^2/day), with T_1 and T_3 being low (69.20 , 33.42 m^2/day) , one S value is zero. Criterion C gives high values for T_4, T_5 (310.08 , 291.85 m^2/day) and two T and three S values are zero. Criterion B gives only two S values as zero and values of T deviate less from their expected values as compared to criterion C or A. Criterion D has no feasible solution. Thus the criterion B gives better results.

If the results obtained with criterion B are compared with those obtained with criterion E it is seen that criterion E gives only one value of T and two values of S as zero as compared to two values of T and two values of S as zero, given by criterion B. Though criterion E gives a high value to T_5 but the number of parameters having values near to approximate values is more with this criterion.

So criterion E gives better results as compared to criterion B. The modified optimization (Table 5.3.2) has no improvement on estimates of parameters.

If weights are used (Table 5.3.3) at d = 0.02 T has all values non-zero and S has zero values for two cells only.

the results obtained with criterion B it is seen that criterion B assigns S value as zero for four cells. But modified optimization (d = 1) assigns only two values of S as zero. For other values of S, the values are nearer to approximate values for modified optimization. T is assigned zero values for 9 cell for both the cases. So modified optimization with d = 1 is better than criterion B.

On analyzing results obtained from Table 5.4.3 i.e. using weights, it is seen that all S values are non zero at d = 1. Four values of T are in permissible limit. If compared with results obtained by modified optimization with d = 1 more values are non zero.

So, overall, modified optimization with weights at d = 1 gives the best results for this case.

5.3 CONCLUSIONS AND SUGGESTIONS FOR FURTHER WORK:

5.3.1 Conclusions:

Following conclusions can be drawn from this study:

(i) The inverse problem can be solved by direct method using criteria A, B, C, D or E. The best results are generally obtained with criteria B, C or E.

(ii) The multicell approach can be used even when some information is missing.

(iii) Modified optimization can be used to improve the criterion E. This method requires assumption of a value for d. d is given arbitrary numbers starting from zero. At d = 0 the criterion is identical to criterion E. As value of d increases the part of the objective function containing d will increase. It has been observed that the values of parameters obtained have considerable variations with small changes in d. For example ,(Table 5.1.2 for the case of 6 cell model with barrier boundaries), the value of d varies from 0.00 to 0.06 but the values obtained for parameters have considerable variations.

(iv) The weights can be applied to improve the results. Weights may depend upon the number of observation wells or area of cell or importance of cell. For instance, the weightage factor applied to the data for different cells can be calculated depending upon

the area represented by the observation wells. If an observation well represent large area, then head data will be less reliable.

5.3.2 Suggestions for Further Work:

(i) In the present study the head data was assumed available for three time periods. This appears to be a limitation. Head data should be taken for a large number of time periods to get good results as it was observed that many values of parameters assume lower boundary values (in the present study zero values).

(ii) A second limitation of the present study is that no positive lower and upper bounds are fixed for S and T values. It is desirable in further work to assign such bounds based on field tests.

(iii) The weights used in the present study depend on the number of observations in a cell. The method is explained in Article 3.7. Other methods of assigning weights should also be tried in further studies.

Table 4.1: Head Data for 6 Cell Model of the Aquifer

Sl. No.	Cell	Peizometeric head $h^k_{i,j}$ in 6 cell model for varying head boundary			
		At the begining of 1st Time Period (m)	At the end of 1st Time Period (m)	At the end of 2nd 2nd Time Period (m)	At the end end of 3rd Time Period (m)
1	1	223.55	221.26	219.87	222.12
2	2	219.94	219.91	218.12	218.95
3	3	216.33	218.56	216.37	215.78
4	4	219.39	219.08	218.26	218.87
5	5	221.28	220.87	220.15	220.98
6	6	223.76	222.66	222.05	223.10
7	FT1	223.30	221.01	219.62	221.85
8	FT2	223.26	222.16	221.55	222.60

Note: For 6 cell model with barrier boundaries on all sides the head data is same as in first 6 cells.

For 6 cell model with constant head boundary the head data in FT1 and FT2 are 223.00 m and 224.00m respectively.

Table 4.2 : Head Data ($h^k_{i,j}$) for 15 Cell, Stepped Aquifer

Sl. No.	Cell (i,j)	At the begining of 1st Time Period (m)	At the End of 1st Time Period(m)	At the end 2nd Time Period(m)	At the end of 3rd Time Period (m)
1	(1,3)	218.35	217.01	219.09	217.48
2	(1,4)	223.55	221.26	219.87	222.12
3	(2,2)	215.24	214.01	212.92	214.13
4	(2,3)	217.49	217.05	217.52	216.92
5	(2,4)	219.94	219.91	218.12	218.95
6	(3,2)	216.89	215.97	213.48	215.72
7	(3,3)	216.63	217.09	215.95	216.35
8	(3,4)	216.33	218.56	216.37	215.78
9	(4,1)	217.01	216.76	215.79	216.41
10	(4,2)	218.32	218.12	218.04	218.06
11	(4,3)	219.15	218.99	218.15	218.71
12	(4,4)	219.39	219.08	218.26	218.87
13	(5,3)	221.68	220.90	220.35	221.06
14	(5,4)	221.28	220.87	220.15	220.98
15	(6,4)	223.76	222.66	222.05	223.10

Table 4.3(a): Average Pumping Rate Per Unit Area ($m^3/day/m^2$)

For Six Cell Model

Cell	Value of $p_{i,j}^{k+1/2}$		
	During Ist Time Period	During IInd Time Period	During IIIrd Time Period
1	0.0	0.0	0.0
2	0.0	0.0	0.0
3	1.7E-3	3.784E-3	1.884E-3
4	1.193E-3	2.780E-3	1.442E-3
5	9.389E-5	1.779E-4	7.453E-5
6	8.900E-5	1.445E-4	4.758E-5

Average Recharge rate/unit area

During Ist Time period $= 2.2208 \times 10^{-4}\ m^3/day/m^2$

During IInd Time period $= 7.7574 \times 10^{-4}\ m^3/day/m^2$

During IIIrd Time period $= 5.5370 \times 10^{-4}\ m^3/day/m^2$

Table 4.3(b): Average Pumping Rate Area for 15 Cell Model (m^2/day/m^2)

Sl.No.	Cell	Value of $p^{k+1/2}_{i,j}$		
		Ist Time Period	2nd Time Period	3rd Time Period
1.	(1,3)	0.0	0.0	0.0
2.	(1,4)	0.0	0.0	0.0
3.	(2,2)	0.0	0.0	0.0
4.	(2,3)	8.464E-4	2.012E-3	1.06E-3
5.	(2,4)	0.0	0.0	0.0
6.	(3,2)	1.746E-4	2.843E-4	9.217E-5
7.	(3,3)	7.272E-5	1.669E-4	8.552E-5
8.	(3,4)	1.700E-3	3.784E-3	1.884E-3
9.	(4,1)	9.699E-4	2.216E-3	1.134E-3
10.	(4,2)	9.471E-5	2.394E-4	1.324E-4
11.	(4,3)	0.0	0.0	0.0
12.	(4.4)	1.193E-3	2.780E-3	1.442E-3
13.	(5,3)	0.0	0.0	0.0
14.	(5,4)	9.389E-5	1.779E-4	7.453E-5
15.	(6,4)	8.900E-5	1.445E-4	4.758E-5

Table 4.4 : Approximate Values of T and S from Field Tests (T:m^2/day)

(1) 6 Cell Model

Cell No.	T	S
1	160	0.118
2	170	0.118
3	150	0.147
4	120	0.140
5	160	0.109
6	140	0.060

(2) 15 Cell Model

Cell	T	S	Cell	T	S	Cell	T	S
(1,3)	160	0.118	(3,3)	160	0.109	(5,3)	160	0.118
(1,4)	160	0.118	(3,4)	150	0.147	(5,4)	160	0.109
(2,2)	170	0.118	(4,1)	240	0.164	(6,4)	140	0.060
(2,3)	130	0.152	(4,2)	170	0.109			
(2,4)	170	0.118	(4,3)	160	0.118			
(3,2)	110	0.069	(4,4)	120	0.140			

Table 4.5: Values of Weightage Factors Assigned to the Cells

6 Cell Model

Cell	Weightage Factor($w_{i,j}$)
1	14.6E-3
2	52.2E-3
3	106.9E-3
4	96.76E-3
5	54.29E-3
6	186.40E-3

15 Cell Model

Cell	Weightage Factor($w_{i,j}$)	Cell (I,J)	Weightage Factor ($w_{i,j}$)	Cell (I,J)	Weightage Factor ($w_{i,j}$)
(1,3)	8.36E-3	(3,3)	43.92E-3	(5,3)	54.7E-3
(1,4)	5.73E-3	(3,4)	41.92E-3	(8,4)	212.75E-3
(2,2)	21.42E-3	(4,1)	30.78E-3	(6,4)	73.03E-3
(2,3)	21.64E-3	(4,2)	72.09E-3		
(2,4)	20.46E-3	(4,3)	339.60E-3		
(3,2)	15.62E-3	(4,4)	37.91E-3		

Table 4.6(a): Coefficients of T and T^2 for Modified Optimization

6 Cell Model

Cell	Coefficient of T2	Coefficient T
1.	3.9E-5	-12.5E-3
2	3.46E-5	-11.7E-3
3	4.44E-3	-13.33E-3
4	6.94E-5	-16.66E-3
5	3.90E-5	-12.50E-3
6	5.10E-5	-14.30E-3

15 Cell Model.

Cell	Coefficient T^2	Coefficient T
(1,3)	3.9E-5	-12.5E-3
(1,4)	3.9E-5	-12.5E-3
(2,2)	3.46E-5	-11.7E-3
(2,3)	3.09E-5	-11.1E-3
(2,4)	3.46E-5	-11.7E-3
(3,2)	8.26E-5	-18.2E-3
(3,3)	3.90E-5	-12.5E-3
(3,4)	4.44E-5	-13.33E-3
(4,1)	1.74E-4	- 8.33E-3

Cell (i,j)	Coefficient of T2	Cell of T
(4,2)	3.46E-5	-11.7E-3
(4,3)	3.90E-5	-12.5E-3
(4,4)	6.94E-5	-16.66E-3
(5,3)	3.90E-5	-12.50E-3
(5,4)	3.90E-5	-12.50E-3
(6,4)	5.10E-5	-14.30E-3

Table 4.6(b) : Values of Coefficients of S^2,S for Modified Optimization

6 Cell Model

Cell	Coefficient of S^2	Coefficient of S
1	71.82	-16.95
2	71.82	-16.95
3	84.16	-13.60
4	51.02	-14.29
5	84.16	-18.35
6	277.78	-33.33

15 Cell Model

Cell	Coefficient of S^2	Coefficient of S
(1,3)	71.82	-16.95
(1,4)	71.82	-16.95
(2,2)	71.82	-16.95
(2,3)	43.29	-13.16
(2,4)	71.82	-16.95
(3,2)	210.04	-28.99
(3,3)	84.16	-18.35
(3,4)	46.28	-13.60
(4,1)	37.18	-12.19
(4,2)	84.16	-18.35
(4,3)	71.82	-16.95
(4,4)	51.02	-14.29
(5,5)	71.82	-16.95
(5,4)	84.16	-18.35
(6,4)	277.78	-33.33

Table 5.1.1 : Results for 6 cell model, with barrier boundaries on all sides (T in m^2/day)

Cell	Criterion									
	A		B		C		D		E	
	T	S	T	S	T	S	T	S	T	S
1	0.00	0.0094	0.00	0.0525	0.00	0.0093	0.00	0.0092	0.00	0.0031
2	268.75	0.0762	244.60	0.0000	157.72	0.0000	78.90	0.0000	133.21	0.0000
3	57.14	0.0553	152.94	0.0404	70.80	0.0252	93.21	0.0000	18.84	0.0451
4	0.00	0.0138	68.90	0.0428	114.69	0.0641	120.80	0.2099	144.60	0.0835
5	304.40	0.0448	242.16	0.0000	131.15	0.0000	209.80	0.0000	237.50	0.0000
6	0.00	0.0262	0.00	0.1007	0.00	0.0245	0.00	0.0432	0.00	0.0324

Table 5.1.2 : Results of 6 cell model, with barrier boundaries on all sides using modified optimization (T: m^2/day)

Cell	d = 0.02		d=0.04		d=0.06	
	T	S	T	S	T	S
1	0.0	0.0017	0.00	0.0005	0.0	0.0001
2	132.58	0.0000	132.44	0.0000	132.55	0.0010
3	66.41	0.0166	66.21	0.0034	66.33	0.0030
4	156.61	0.0181	159.58	0.0000	159.40	0.0000
5	238.83	0.0000	240.38	0.0000	240.30	0.0000
6	0.00	0.0049	0.00	0.0004	0.00	0.0000

Table 5.1.3 : Results of 6 cell model, with barrier boundaries on all sides using weights

Cell	d = 0.0		d=0.002		d=0.004		d=0.006		d=0.008	
	T	S	T	S	T	S	T	S	T	S
1	0.00	0.0089	0.000	0.0080	0.000	0.0083	0.000	0.0045	0.000	0.0030
2	260.00	0.0000	263.150	0.0000	265.300	0.0000	266.700	0.0000	271.400	0.0000
3	15.69	0.0437	14.670	0.0388	137.900	0.0317	133.800	0.0240	0.180	0.0170
4	123.59	0.0983	124.020	0.0854	124.600	0.0617	125.360	0.0492	147.370	0.0332
5	180.54	0.0000	178.680	0.0000	177.500	0.0000	177.750	0.0000	154.620	0.1716
6	0.00	0.0248	0.000	0.0157	0.000	0.0074	0.000	0.0028	0.000	0.0070

Table 5.2.1 : Results of 6 cell model, having varying head boundaries on two sides and barrier boundaries on reamining sides (T: m^2/day)

Cell	Criterion									
	A		B		C		D		E	
	T	S	T	S	T	S	T	S	T	S
1	0.00	0.0185	0.00	0.0657	0.00	0.0085	0.00	0.0119	0.00	0.0056
2	235.00	0.0318	220.60	0.0000	2.24	0.0000	69.19	0.0000	97.80	0.0000
3	97.47	0.0518	145.80	0.0337	149.68	0.0548	89.34	0.0000	80.95	0.0455
4	0.00	0.0865	74.00	0.2187	114.68	0.0467	120.81	0.2110	150.66	0.0810
5	201.44	0.0518	253.50	0.0000	131.15	0.0000	202.25	0.0000	243.54	0.0000
6	0.00	0.0000	0.00	0.1076	0.00	0.0245	0.00	0.0393	0.00	0.033
FT1	0.00	0.0000	34.86		1277.7		0.00		137.20	
FT2	209.00	0.0000	0.00		0.0		0.00		0.00	

Table 5.2.2 : Results of 6 cell model having varying head boundaries on two sides and barrier boundaries on remaining two sides using modified optimization method (T: m^2/day).

Cell	d = 0.04		d=0.06		d=0.08	
	T	S	T	S	T	S
1	0.00	0.0019	0.00	0.0004	0.00	0.000
2	95.77	0.0000	97.79	0.0005	97.59	0.0001
3	78.49	0.0067	78.51	0.0067	78.49	0.0067
4	106.12	0.0000	164.32	0.0000	164.23	0.0000
5	246.82	0.0000	245.57	0.0001	245.43	0.0000
6	0.00	0.0027	0.00	0.0001	0.00	0.0000
FT1	132.02		101.47		101.41	
FT2	0.00		0.00		0.00	

Table 5.2.3 : Results for 6 cell model with varying head boundaries on two sides using weight(T:m^2/day)

Cell	d=0		d=0.002		d=0.004	
	T	S	T	S	T	S
1	0.00	0.0089	0.00	0.0070	0.00	0.0063
2	260.90	0.0000	263.17	0.0000	265.30	0.0000
3	15.69	0.0437	14.67	0.0380	13.79	0.0317
4	123.60	0.0983	124.02	0.0780	124.65	0.0677
5	180.56	0.0000	178.69	0.0000	177.55	0.0000
6	0.00	0.0248	0.00	0.0150	0.00	0.0074
FT1	0.00				0.00	
FT2	0.00				0.00	

Table 5.3.1 : Results for 6 cell model with constant head boundary on two sides, and barrier boundaries on remaining sides (T, m^2/day)

Cell	Criterion									
	A		B		C		D		E	
	T	S	T	S	T	S	T	S	T	S
1	69.20	0.0234	0.00	0.0632	0.00	0.00	No feasible solution		0.00	0.0035
2	342.34	0.0000	216.78	0.0000	105.48	0.00			81.33	0.0000
3	33.42	0.0506	147.14	0.0346	0.00	0.0564			57.68	0.0463
4	0.00	0.0976	75.99	0.2200	310.03	0.0463			269.92	0.0431
5	151.31	0.0517	256.70	0.0000	291.85	0.0000			326.83	0.0000
6	0.00	0.1290	0.00	0.1094	158.28	0.0099			153.76	0.0090
FT1			0.00		0.00				0.00	
FT2			0.00		216.22				265.59	

Table 5.3.2 : Results for 6 cell model with constant head boundary on two sides and barrier boundary on remaining sides solution using modified optimization (T, m^2/day)

Cell	d=0.01		d=0.02		d=0.03	
	T	S	T	S	T	S
1	0.00	0.0021	0.00	0.010	0.00	0.0004
2	77.33	0.0000	76.60	0.000	76.88	0.0000
3	52.23	0.0273	52.09	0.0118	52.64	0.0040
4	294.27	0.0104	298.76	0.000	297.96	0.0000
5	330.73	0.0000	333.96	0.000	334.47	0.0000
6	177.13	0.0008	181.24	0.001	180.82	0.0000
FT1	0.00	0.00	0.0		0.0	
FT2	265.59	268.70	272.94		273.62	

Table 5.3.3: Results for six cell model with constant head boundary on two sides and barrier boundaries on remaining sides (T, m^2/day) (Using weights)

Cell	d = 0.0		d=0.010		d=0.020		d=0.030		d=0.040	
	T	S	T	S	T	S	T	S	T	S
1	404.50	0.00	404.50	0.0051	404.50	0.0022	404.5	0.0007	404.50	0.0007
2	44.90	0.00	142.00	0.0000	141.50	0.0000	141.3	0.0000	141.39	0.0000
3	4905.00	0.00	13.40	0.0269	11.15	0.0117	13.83	0.0039	13.83	0.0039
4	470.90	0.00	273.80	0.0820	282.47	0.0044	279.02	0.0628	279.02	0.0628
5	287.50	0.00	237.80	0.0742	237.20	0.1313	243.90	0.0710	243.85	0.0710
6	44.00	0.00	231.60	0.00	129.40	0.00	136.50	0.0000	136.52	0.0000
FT1	44.00		44.00		44.90		12.44		44.00	
FT2	47033.00		140.00		139.20		148.00		148.00	

Table 5.4.1 : Results for 15 cell mode, with barrier boundaries on all sides (T: m^2/day)

Sl. No.	Cell (I,J)	Criterion A'		B		E	
		T	S	T	S	T	S
1	(1,3)	0.00	0.1526	0.00	0.0251	0.00	0.0053
2	(1,4)	626.21	0.1038	0.00	0.0000	0.00	0.0114
3	(2,2)	0.00	0.0000	0.00	0.0992	0.00	0.0000
4	(2,3)	0.00	1.6296	0.00	0.1170	0.00	0.0000
5	(2,4)	0.00	0.3731	141.37	0.0000	180.75	0.0000
6	(3,2)	0.00	0.0424	0.00	0.3441	0.00	0.0000
7	(3,3)	481.45	0.6108	242.97	0.0000	0.00	0.0000
8	(3,4)	0.00	0.3754	0.00	0.1777	1.39	0.0454
9	(4,1)	810.00	0.0000	231.03	0.4653	194.82	0.0818
10	(4,2)	708.81	6.7900	0.00	0.1662	0.00	0.2121
11	(4,3)	0.00	1.4130	0.00	0.0397	15.85	0.0000
12	(4,4)	0.00	0.00	144.81	0.0559	18957.0	0.0807
13	(5,3)	155.96	1.3394	153.83	0.0835	0.0	0.0000
14	(5,4)	103.14	1.2800	176.96	0.4180	219.20	0.0000
15	(6,4)	26.43	0.9314	0.00	0.0000	0.00	0.0300

Table 5.4.2: Results for 15 cell model having barrier boundary on all sides, using Modified optimization (T: m^2/day)

Sl. No.	Cell (I,J)	d = 1		d=2		d=3	
		T	S	T	S	T	S
1	(1,3)	0.00	0.0053	0.00	0.0053	0.00	0.0053
2	(1,4)	0.00	0.0090	0.00	0.0000	0.00	0.0000
3	(2,2)	0.00	0.0028	0.00	0.0000	0.00	0.0000
4	(2,3)	0.00	0.0000	0.00	0.0000	0.00	0.0000
5	(2,4)	176.70	0.0070	173.37	0.0001	171.69	0.0000
6	(3,2)	0.00	0.0003	0.00	0.0000	0.00	0.0000
7	(3,3)	0.00	0.0067	0.00	0.0001	0.00	0.0000
8	(3,4)	1.38	0.0454	1.39	0.0454	1.39	0.0454
9	(4,1)	194.14	0.0818	189.57	0.0818	182.91	0.0818
10	(4,2)	0.00	0.0361	0.00	0.0004	0.00	0.0000
11	(4,3)	20.64	0.0098	20.44	0.0001	20.24	0.0000
12	(4,4)	199.26	0.0807	196.13	0.0807	192.24	0.0807
13	(5,3)	0.00	0.0039	0.00	0.0001	0.00	0.0000
14	(5,4)	220.52	0.0013	217.63	0.0000	214.67	0.0000
15	(6,4)	0.00	0.0000	0.00	0.0000	0.00	0.0000

Table 5.4.3: Results for 15 cell model with barrier boundaries on all sides, using weights (T: m^2/day)

Sl. No.	(Cell)	d = 0 T	d = 0 S	d=1 T	d=1 S	d=2 T	d=2 S	d=3 T	d=3 S
1	(1,3)	0.00	0.0052	0.00	0.0052	0.00	0.0052	0.00	0.0052
2	(1,4)	0.00	0.0186	0.00	0.0010	0.00	0.0000	0.00	0.0000
3	(2,3)	0.00	0.0000	0.00	0.0028	0.00	0.0000	0.00	0.0119
4	(2,3)	0.00	0.0119	0.00	0.0119	0.00	0.0119	0.00	0.0000
5	(2,4)	271.39	0.0052	272.25	0.0720	267.90	0.0001	265.26	0.0000
6	(3,2)	0.00	0.0000	0.00	0.0002	0.00	0.0000	0.00	0.0000
7	(3,3)	98.26	0.0000	94.21	0.0066	91.46	0.0001	0.00	0.0510
8	(3,4)	0.00	0.0510	0.00	0.0510	0.00	0.0510	90.19	0.0861
9	(4,1)	169.00	0.0861	160.87	0.0861	155.42	0.0861	0.00	0.0000
10	(4,2)	0.00	0.1787	0.00	0.0355	0.00	0.0004	154.01	0.0000
11	(4,3)	0.00	0.0000	0.46	0.0100	0.0896	0.0001	0.00	0.0968
12	(4,4)	136.71	0.0968	137.36	0.0968	134.94	0.0968	0.1946	0.0000
13	(5,3)	0.00	0.0000	0.00	0.0039	0.00	0.0001	132.40	0.0000
14	(5,4)	183.71	0.0000	182.85	0.0013	180.46	0.0001	0.00	0.0251
15	(5,4)	0.00	0.0251	0.00	0.0251	0.00	0.0251	177.90	0.0000

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